Lisser Rye Ejersbo

Design and redesign of an in-service course: The interplay of theory and practice in learning to teach mathematics with open problems

PhD Dissertation

Learning Lab Denmark
Danish School of Education, University of Aarhus

Design and redesign of an in-service course: The interplay of theory and practice in learning to teach mathematics with open problems

PhD Dissertation in Mathematics Education
By Lisser Rye Ejersbo

Supervisor: Mogens Niss

Photos: Lisser Rye Ejersbo

Total numbers of pages: 204

June 2007

Learning Lab Denmark
Danish School of Education
Aarhus University
Tuborgvej 164
2400 København NV
Email: lre@dpu.dk

Table of contents:

Acknowledgements

This dissertation has been written at Learning Lab Denmark, The Danish University of Education (DPU). I would like to thank Hans Siggaard Jensen, DPU, and the 'Science, Technology and Learning' group for providing an inspiring workplace; special thanks go to my colleagues during the whole study Robin Engelhardt, Morten Misfeldt, and Mette Andresen for inspiration and support and to Nicolai Paulsen, DPU, for editing my English.

Especially, I want to thank my supervisors. In the first half of my study Jeppe Skott, DPU, and Benny Karpatschof, KU, did a great job with supervising me, and in the last part Mogens Niss, RUC, was an excellent supervisor; I can't thank him enough for that.

I have received valuable assistance from several sources during the course of writing this dissertation. I would like to thank Andy DiSessa and Uri Leron for inspiring discussions and good advices, Barbro Grevholm and Anna Kristiansdottir for their hospitality when I visited Högskolan in Agder, and Mike Askew for hosting my one-semester visit to King's College London.

The study and the project behind the dissertation involved many teachers and pupils, whom I found through the support I got from CVU, Copenhagen and North Zealand; I would like to thank all of them for their willingness to cooperate. Especially, I would like to thank the teachers and the pupils for the work we did together and for all the insights that I gained through that work.

There are of course many others who helped with various aspects of this work through suggestions and discussions. Although I cannot list them all, they too deserve my thanks.

Last but not least I want to thank my family and good friends for being patient, supportive and encouraging the whole way through.

Lisser Rye Ejersbo, June 07

Introduction

Teaching mathematics with open problems is both vital and difficult. And, as I will demonstrate in this dissertation, much of the difficulty stems from the need to master, beside the mathematical skills, also the skills of communication and reflection. This study centres on an in-service course for Danish teachers, whose main goal has been to develop competencies in using open problems to teach and assess mathematics in final exams. The Danish previsions for the final, school leaving oral exam stipulate that it should include a practical problem with an open-ended approach, and that the assessment will be based on a group of one to three pupils solving and explaining a solution in a time schedule of two hours. The in-service course was originally designed and carried out by me in 1996 to meet the Danish departmental executive order about the new oral group examination in mathematics after the ninth and tenth grades. In the following years, more than six hundred mathematics teachers participated in repetitions of the course. The stages of the course that were subject of my research took place from 2002 to 2006, with the resulting strengths and weaknesses of the researcher and teacher educator being the same person.

In evaluating my own course, I first relied on the teachers' feedback, which was generally very positive. However, when I later started to make observations in their classrooms, I realised that my intentions were less than fully implemented in their teaching. This is particularly true of the important but difficult skills of communication and reflection. While there could be many reasons for why this was so, I decided to concentrate on redesigning and improving 'my' course. The subsequent development of the in-service course went through various cycles of instructional design, inspired by specific theoretical concepts from the research literature, and 'transposed' by me into instructional practice.

The present dissertation consists of three parts: PART A: 'Theoretical Preliminaries' comprising three chapters, PART B: 'Pre-Study' comprising seven chapters and PART C: 'Main Study' comprising five chapters.

In the theoretical preliminaries, the first chapter is a description of the Danish school system, and the conditions for pre- and in-service education. The second chapter presents my research background and the research questions, while the third chapter comprises a description of my research methodology adopted for this dissertation.

Part B is a pre-study comprising the studies of the participating teachers before, during and after an in-service course held in 2002: Expectations and teaching habits of the participating teachers, reactions to the course, their use of tools from the course in their subsequent teaching, and finally their pupils' responses.

The main study, part C, comprises a theoretical look at what I call 'meta-didactical transpositions', i.e. the researchers' adaptation of theoretical concepts from the research literature into the practice of an in-service course, combined with the various practical cycles of redesigning the courses during 2004-6 with an emphasis on communication and reflection related to mathematics.

The 'meta-didactical transpositions' in this study are employed through a set of guiding principles for the teaching methods. The methodology used in the investigation is design research, where video- and audiotapes from the in-service courses are used as data along with logs written by the participating teachers. My focus is on the redesign of the instruction and the teachers' reactions when they worked with the new design. Most mathematics teachers do not read research literature, and in-service education seems a natural place for them to meet theoretical concepts.

Unfortunately, practice and theory often have difficulties benefiting from each other; in the redesigned part, I try to transpose theoretical concepts into practice and, conversely, let practice inspire new theoretical ideas.

PART A: Theoretical Preliminaries

1. Danish ministerial provisions for education

1.1 Provision for education in the 'Folkeskole'

In Denmark, the regulations for education in the 'Folkeskole' (the municipal primary and lower secondary school, K-tenth grades) result from a mixture of centralized and decentralized decision making. The following more general regulations are from the homepage of the Danish Minister of Education, www.uvm.dk (quotations are all in italic):

> *The Danish Folkeskole is centrally regulated by the Act on the Folkeskole, which sets the framework for the activities of the school. This means that all municipal schools have common aims, common provisions for the subjects that are to be taught at the different form levels, common provisions for the central knowledge and proficiency areas of the subjects and common provisions for the organization of the school system. But it is the responsibility of the individual municipality to decide how the schools of the municipality are to function in practice within the framework of the Act. (…)*
> *The central administration of the Folkeskole is in the hands of a department in the Ministry of Education. The Danish Parliament takes the decisions governing the overall aims of the education, and the Minister of Education sets the targets for each subject. But the municipalities and schools decide how to reach these targets.*
> *The Ministry of Education publishes curriculum guidelines for the individual subjects, but these are seen purely as recommendations and as such are not mandatory for local school administrators. Schools are permitted to draw up their own curricula as long as they are in accordance with the aims and proficiency areas laid down by the Minister of Education. However, nearly all schools choose to confirm the centrally prepared guidelines as their binding curricula. (Official translation)*

The Folkeskole is a universal school, where classes are formed based on the age of the pupil, and not on the basis of the subject-specific proficiency of the pupils. The Folkeskole provides most of the basic schooling in Denmark. Denmark also has private independent schools, so-called free schools, which offer teaching that more or less equals that of the Folkeskole, but the framework for the organisation of the teaching is less restrictive. It is laid down in the annual Finance Act that the State's operational grant to the free schools is 85% of the expenses per pupil. In 2000, 11.5% of the grade school pupils attended free elementary schools. In 1999, the teacher/pupil ratio in the Folkeskole was 1:10.7 against a ratio of 1:9.7 at the free schools. (Ministry of Education, 2002), p.5 and p.52)

In 1993, the Danish Parliament passed a new Folkeskole Act; and in 1997, a new ministerial order for the final, school leaving mathematical examination was introduced. According to this order, the mathematical examination consisted of two written and one oral examination. Standard rules for all examinations are meant to ensure uniformity throughout the country. For the same reason, the papers for the written examinations are set and marked centrally, while the oral examinations are set and marked locally with the involvement of external examiners. Examinations are not compulsory for the pupils. The pupil is free to decide whether or not to sit for them after consultations with the school; in practice this means his or her own teachers together with the parents. Despite it being voluntary, almost every pupil chooses to sit for the examination (Danmarks Evalueringsinstitut, 2002).

The hierarchy of the curricular decisions for mathematics is outlined in the following table (my translation and formulation):

Level 1: Centralized decisions, binding for teachers and municipalities:	The Education Act Central knowledge and proficiency areas for mathematics Departmental orders for the examinations
Level 2: Decentralized decisions, bindings for the teachers if not changed by the individual municipality:	Curriculum guidelines for mathematics The syllabus with subjects, activities and working forms
Level 3: Left to the teachers' decisions and not binding:	Teaching recommendations from the ministry

Figure 1: The hierarchy of the curricular decisions for mathematics

1.2 Centralized decisions
(Binding for the teachers and the individual municipality)

The Aims of the 'Folkeskole' are the overall objectives in the Danish nine-year compulsory grade school, where mathematics is taught at all grades with recommended four periods per week. Pupils are normally taught in classes which remain together throughout the entire course in the Folkeskole. The whole class follows the same basic curriculum in mathematics, which means that any differentiated individualised teaching must necessarily take place within the framework of the class.

The ministerial aims of the mathematics education are formulated in the following list of 'what', 'how' and 'why'. The non-italic parts (my choice) show the expectation of the pupils' outcome of the school mathematics:

> *It shall be the aim of the teaching in the subject of mathematics that the students become able to understand and use mathematics in contexts relating to everyday life, social life and natural conditions. Analysis and argumentation shall form part of the work with topics and problems.*
> *The teaching shall be organised so that the pupils build up mathematical knowledge and proficiency on the basis of their own prerequisites. The pupils shall, independently and together, learn that mathematics is both a tool for problem-solving and a creative subject. The teaching must give the pupils a sympathetic insight and further their imagination and curiosity.*
> *The teaching must ensure that the pupils experience and realise the role of mathematics in a cultural and social context. With a view to enabling the pupils to take responsibility and exert influence in democratic solidarity, they will be able to relate in an appraising manner to the use of mathematics. (Official translation from uvm.dk)*

As for differentiation, we find in § 18 that:

> *The organisation of the teaching, including the choice of teaching and working methods, teaching materials and the selection of subject-matter, shall in each subject live up to the aims of the Folkeskole and shall be varied so that it corresponds to the needs and prerequisites of the individual pupil. (Official translation)*

Paragraph § 13.2 reads:

As part of the teaching, there shall be a regular assessment of the pupils' benefit from the teaching. The assessment shall form the basis for the guidance of the individual pupil with a view to the further planning of the teaching. (Official translation)

In other words: The Danish teacher has to prepare his or her teaching to meet the needs of every individual pupil guided by a regular assessment of the pupils (from now on his or her will be used randomly for researcher and teachers, if the gender is not known for certain).

1.3 Provisions for the oral exam

The ministerial order for the oral mathematical examinations reads excerpts from §29 date 27.01.97:

No.5. At the oral part of the examination, each of the following areas should be presented for the options:
a) Work on numbers and algebra
b) Work on geometry
c) Applications of mathematics
d) Communication and problem solving
No.6. The examination may be individual or in groups of two or three pupils. The examination should be set up to last two hours, where some six pupils work simultaneously in the same room. The marks are decided and given at the end of the same period.
No.7. The examination is based on a task built on practical problems. The task must offer the pupils an opportunity to use mathematical methods, knowledge and skills, and let them demonstrate the application of mathematics through investigation, systematizing and argumentation. The pupils should switch between theory and practice in their work. Pupils are allowed to use the same aids as used in the day-to-day teaching. A computer must be available. While the pupils work, the teacher and the external examiner talk with the group and the individual pupils. A final clarifying talk about the practical and theoretical considerations will end the examination.
No.8. The examination is aimed to determine whether the pupil is able to use the mathematical terminology and working methods in arguing for the results obtained, choice of procedure, and knowledge about the mathematics. The pupils are assessed individually and receive individual marks. (My translation)

1.3.1 A change in 2005

From 14 July 2005, a new ministerial order - from a new government - changed the oral examination to be individual only, although it allowed for about four pupils working simultaneously in the same room with different tasks for about an hour. The specification for the task is still that it should be based on a practical problem with an explicit mathematical focus; open-ended tasks are acceptable. The pupil can still choose different ways or models to solve the problem. This charge would seem to spell disaster for the research project, but this has not proved to be the case. In the study, I focus on the day-to-day teaching, where the teachers prepare themselves and their pupils for work with open practical problems to meet the requirements for the oral examination. Still, it is recommended that cooperation between pupils in the mathematical classroom will train them to practice the new oral exam. At the in-service course in 2005, I ran the course under the new rules. How to teach and organise work with open practical problems is still an important and necessary issue to practice, related to the provision of the oral exam. For the in-service course in 2006, twenty teachers enrolled, which shows the continue relevance and demand for the course.

1.4 The central knowledge and proficiency areas (Binding for the teachers if not changed by the individual municipality)

The regulations are written in the official so-called subject booklet (Faghæfte 12, 2003, latest edition) for mathematics. The syllabus comprises the following four topics:

a) Work on numbers and algebra
b) Work on geometry
c) Applications of mathematics
d) Communication and problem solving

For each grade, it is spelled out how these four bullet points can be transformed to 'mathematics-for-teaching'.

The booklet from the Ministry of Education about the school leaving examination (Undervisningsministeriet, 1998) (Mathematics Examinations, ministerial order and recommendation, the Ministry of Education, revised in 1998, original edition is from 1996) includes a passage in which it is described that the practical problems for the oral exam must contain a mix of the above-mentioned four areas and be formulated in such a way, so that the pupils can work with a problem on several levels. Furthermore, the teacher must produce enough questions that the last group examined has at least four choices.

1.5 Recommendations for the teachers and textbooks

In terms of curricular decision making, the recommendations in the Faghæfte 12 (2003) are a collection of teaching/learning materials, teaching methods and assessment activities. It is difficult to interpret what is recommended and what is binding, and how to transform it to every day practice. The following examples show this:

> *The mathematics teacher is, to a large extent, to select the mathematical contents within the framework of the syllabus in such a way that it fulfils the requirements in the aims of mathematics instruction within it. (p.65)*
> *The description in the syllabus builds on applications of mathematics, and on the subject being based on daily life, practical connections, and problems. When choosing the mathematics content it is not enough to argue that it is good for mathematics itself, rather the arguments have to come from applications of the subject or from other subject areas. (p.66)*
> *When choosing the textbook, it is important to carefully consider the didactic approach in the particular textbook under consideration; whether it is in agreement with the syllabus. (p.67)*
> *The Act requires teachers to take into consideration the needs and present competences of the individual pupil. Therefore the teacher has to negotiate learning goals with the individual pupil on a regular basis. (…) Teaching is the teacher's responsibility. Consequently the teacher sets the objectives. When so doing, the teacher must of course consider the responsibilities and goals of the students: To what extent and in what ways does the student become involved in this particular instructional sequence? (p.68)*
> *Learning happens through the pupils' activity; therefore preparation of the teaching should take this into account. (p.69) (My translation)*

We had, in 2005, eleven different mathematics textbook systems in Denmark. The Danish government is not involved in questions about textbooks. Textbooks are published under market conditions; publishers decide which books they want to publish. Schools are free to decide which textbooks they want to buy and use methods financial constraints. These circumstances influence both the content and the wrapping of the textbook, and means that the content must be easy to follow for the experienced mathematics teachers as well as for the less experienced ones and the book must present itself in an inviting way to be sold. It is difficult for the teachers to choose the best textbook system and see through the marketing hype; often it is the economical circumstances of the school and not the individual teacher that determine what is being bought. This means that if the teacher finds that the actual textbook is not in agreement with the syllabus, it is the teachers' responsibility to make up for what is missing from the textbook, so that the teaching is in accordance with the syllabus. And with regards to the application of mathematics, many mathematics teachers do not have knowledge or experiences about this topic.

The textbook plays a crucial role for the mathematics teaching. An investigation about mathematics in the Folkeskole (Danmarks Evalueringsinstitut, 2002) shows that most of the pupils experience their mathematics teaching as traditional, monotonous and old-fashioned, while the teaching consists of only using a text book. It is not easy to find new tasks or ways to vary the daily teaching. I know this because prior to the in-service course in 2002, I invited the enrolled teachers to participate in this study and asked the volunteers to let their pupils solve some different tasks I sent them. The three tasks are different in their degree of openness: one is closed but the way to solve it is open, one has some degree of openness in the solution, and one is very open with much information and decisions for a solution left to the pupils.

I received 144 pupils' solutions and questionnaires. The result of the answers clearly demonstrated that the pupils were not used to work with open problem solving; all the pupils supplied the same answers. Whether they liked the tasks differed from class to class. Almost everybody liked the first task; it was easy, of a familiar type and didn't take long to solve. Half the pupils didn't like the new open tasks, while the rest expressed interest and a liking for the open style. It turned out that the pupils who liked the tasks generally came from the same classes, and like vice with the pupils who didn't like the open tasks. My interpretation of this observation is that we catch a glimpse of a 'didactical contract' (Brousseau, 1997) between the teacher and the pupils in the respective classes. I will not go further into the results other than highlight the need for the teachers to be inspired in their work with open practical problems, which is the main topic on the in-service course.

1.6 Teaching methods

In Denmark, we have a long tradition for freedom in choosing teaching methods. In a ministerial order about the objectives for the Folkeskole from 1941 (Bekendtgørelse om Maalet for Folkeskolens Undervisning, af 24 maj 1941) we find:

> *(…) no attempts should be made to lead the teaching in specific directions,as log as the overall goals are achieved. The Ministry does not want to give guiding principles for how the frames should be filled in (…) (p. 5) (My translation)*

This is repeated in the official teaching recommendations from 1961 (Den blå betænkning II, 297):

> *The teacher has the right to choose teaching methods that fit his qualifications, and which take into account the requirements from the teaching recommendations. (p.15) (My translation)*

> *In 1975: …the choice of the teaching methods and aims should be made in cooperation between the teacher and the pupils. (p.25) and in 1993 (p.33): …the methods should be differentiated in such a way that it matches each individual pupil's need and understanding. (My translation)*

This degree of freedom has been a part of the Danish teachers' tradition in decades, but the wording has changed. Stricter requirements force the teachers to make less predictable curricular judgments in the classroom, which may bring them into a kind of limbo. Skott (Skott, 2004) calls this phenomenon 'forced autonomy' and describes it as a manoeuvre the teacher is required to perform independently and autonomously in order to sustain individual and collective learning opportunities through decision-making on-the-spot. Bateson (Bateson, 1972) calls such forces 'double bind processes'. These forces gives rise to an impossible situation, where more powerful persons require other people to do activities voluntarily. Forcing people to do the actions voluntarily is obviously a contradiction. Nordenbo (Nordenbo, 1997) describes how the degree of freedom for the Danish teachers has changed from being based on pedagogical principles to being

> *(…) interpretations of the intention of the political decisions (…) of a subject syllabus and to investigate how these political decisions can be realized in an appropriate way. (p.162) (my translation)*

In my in-service teaching, I encounter the same needs: The teachers ask for help to understand what the ministerial orders mean and how to interpret and transpose them into useful methods. The teachers want to play by the rules and mention parents, pupils and official tests as control systems. They seem to be afraid to do it incorrectly, which is why it is important to interpret the regulations in an unambiguous manner. Metaphorically speaking, we could call the teachers front soldiers, and if they fail, the responsibility is all theirs and they will be penalised. How I deal with these needs on the in-service course will be discussed later.

In the booklet 'Mathematics Examinations', it isn't quite clear what is mandatory for the teacher or meant as recommendations only, or what it is up to the local municipality to decide. The following could be construed as either binding or not binding.

> *One way to ensure that [that the presentation provides pupils an opportunity to work on several levels] is to make the presentation open.*
> *A presentation can be made to be open in several ways, e.g. by:*
> - *The way the task is described in the initial presentation,*
> - *Presentation of several different methods to solve the task,*
> - *Presentation of several different strategies to solve the task, and*
> - *Ensuring that tasks have several possible and equally correct solutions. (p. 27) (my translation)*

In the booklet, it is further explained that a practical problem could be a situation from daily life such as political poll, a skiing holiday etc. The problem might also be something specific and hands-on like a cube, or it might be something the pupils produce themselves during the examination, such as a physical model. The openness is characterized by the opportunity to work on different levels. The different between methods and strategies is never explained and it is not obvious what it means, other than both are parts of the process.
The communication during the examination is described like this in the booklet:

> *The nature of the questions the teacher or the external examiner ask and the situated communication created during the examination is crucial for the course of the oral exam. A question can be either thought of as evaluating or push the examinant's work ahead. Furthermore, the pupils should be able to understand the questions asked by the external examiner, who may use a different jargon than their own teacher. (p.27) (my translation)*

The assessment should be based on how the pupils apply concepts and working methods in relation to the formulated practical problem. But again, the teachers express stress and confusion about how to interpret the ministerial orders and about how to transform the recommendations into a mathematics teaching practice they can manage and be satisfied with.

1.7 Pre-service education in Denmark

In Denmark we have at present:

18 colleges of education throughout the country offer teacher education. The colleges train teachers for the entire "Folkeskole". Denmark has a unified teacher training system for the whole period of compulsory schooling. A number of features are particularly characteristic of the Danish system, the most salient of these being the broadness of the curriculum, the in-depth study of four school subjects and the integration between theory and practice that exists between didactics, psychology, etc., school subjects and teaching practice.(…) The duration of training is 4 years, including 24 weeks of teaching practice. (From www.uvm.dk) (official translation)

This regulation with its 'in-depth study of four school subjects' was passed in 1997. One of the four subjects must be either mathematics or Danish. Every teacher is educated to teach for the whole span of compulsory schooling, which means from grades 1 to 10. In Danish schools, teachers often work in teams and classes usually have only a few teachers. One consequence of this is that many teachers teach mathematics in spite of the fact that they have not been specifically trained to do so. In primary school, only 40 % of the mathematics teaching is handled by teachers specialised in mathematics, while in lower secondary classes about 75% of the mathematics teachers are specialised in mathematics (Undervisningsministeriets nyhedsbrev (Newsletter from the ministry of Education) nr. 15, 2002, www.uvm.dk,). At the in-service course I have studied, the participating teachers all teach in lower secondary classes. Applying statistics above, 75% of 'my' participating teachers should therefore be specialised in mathematics. In practice, however - from my own in-service experiences - more than 25% of 'my' participants are not specialised in mathematics. The reason could be the schools' inclination to send those teachers to the course, who teach mathematics in lower secondary classes without being specialised in mathematics, even though this is a course, where it is taken for granted that the teachers are well trained to teach lower secondary classes in mathematics. The specialised mathematics teachers have furthermore different basis educations caused by the changes in pre-service education during the years; therefore, the total group is educationally a very mixed group.
An overview of changes in the educational regulation is showed in the following table, where a lesson corresponds to 45 minutes:

Resources: The year:	Compulsory basic education, that allows teaching in primary school	Specialisation (to be chosen), that allows teaching in lower secondary classes	Total
Educational Act from 1966, enacted in 1969	140 lessons (about 0.34)	392 lessons (about 0.96)	532 lessons (about 1.31 of one person's work in one years)
Educational Act from 1991, enacted in 1992	0.36 of one person's work in one year	0.60 of one person's work in one year	0.96 of one person's work in one year (about 391 lessons)
Educational Act from 1998, enacted in 1998		0.70 of one person's work in one year (about 285 lessons)	0.70 of one person's work in one year

Figure 2: An overview of changes in the educational regulation for mathematics in pre-service from 1966-2006 (Hansen, 2006)

It is legal for the institution to choose the number of lessons for 'one persons work in one year' themselves. Hence there are differences from one institution to another, and the number of lessons cannot easily be compared. In one pre-service institution, 0.70 of one person's work in one year meant 285 lessons of 45 minutes. From august 2007 a new regulation for pre-service mathematics reads 72 ECTS points, which come up to a total of 480 lessons (1.18 of one person's work in one year).

The participating teachers at the in-service course were a mixed group of teachers with varying levels of mathematics education. Some come from Folkeskolen and some from free elementary schools.

1.8 In-service education in Denmark

In-service education has a relative long history in Denmark as it dates back to 1856 (Christiansen, 1990), (Rigbolt, 2000). Any teacher in primary and lower secondary has the opportunity to participate in further education courses, throughout their professional life, provided their school allows it, but they have no obligation to do so and their salary does not depend on the number of in-services courses they take. This means that the teacher's participation is normally voluntary, subject to the condition that the school allows them to participate in the course and pays both for the course and for the time the teacher spends on the course.

Until the 1980's, in-service education was offered almost exclusively by the Royal Danish School of Educational Studies. The in-service courses were based on the vision that teachers of any subject should acquire knowledge and insight concerning new developments for the teaching of their particular subjects. The courses were only directed and open for 'teachers-in-service', which could be teachers with only little professional experience through to very experienced teachers. A yearly syllabus listed the different in-service courses offered and the teaching in the courses had to be based on research. What exactly 'based on research' meant is difficult to pinpoint; it seems that the sole criterion was that the teacher educator had to be employed at a research institution, but not necessarily as a researcher. In the 1990's this monopoly was broken, and new institutions began to offer in-service courses for teachers; in these institutions teaching was not required to be research-based. At the Royal Danish School of Educational Studies, a department consisting of pedagogical consultants was established in 1992. These consultants were not necessarily researchers, but sometimes 'only' especially capable teachers. Yet, the teacher-students liked the consultants' in-service courses, which were also cheaper than the 'research-based' ones conducted by researchers. In-service courses became a commodity on the market.

In 2000, the Royal Danish School of Educational Studies merged with two other institutions to become the Danish University of Education (DPU) (From July 2007 DPU becomes 'The Danish University school of Education' and a part of Aarhus University). The Royal Danish School of Educational Studies's core activity was educational and didactical research and making the results of scientific research accessible to teachers through in-service courses. The DPU does not solely serve teachers and its goal is

> *committed to the pursuit of excellence in teaching and research, and to promote research and postgraduate education. (From: www.dpu.dk)*

To address the dilemma that the teachers still needed in-service education, a new type of institution was created in 2001: Centres for Further Education (CVU). The pedagogical consultants from the Royal Danish School of Educational Studies were moved to those centres. A CVU comprises different educational institutions that teach as far as to a bachelor’s degree – the students include prospective teachers, nurses, kindergarten teachers etc. Those Centres continue the tradition from the Royal Danish School of Educational Studies and each of them publishes a catalogue with the in-service courses sold next year. The difference is that all the in-service courses are now given on market conditions, and the courses are designed and run mostly by pedagogical consultants, who could be just regular teachers or researchers.

2. Research Background

2.1 Personal background

In 1996, I was employed as a pedagogical consultant at the Royal Danish School of Educational Studies to organise and carry out in-service education in mathematics and didactics of mathematics. I know the discussion about the confusion of the expression ’didactics of mathematics’ and ’mathematics research’, but in this dissertation, I chose to use the expression ’didactics of mathematics’ for what in English is mostly called ’mathematics education’

In my design and running of courses as a pedagogical consultant, I drew on my experiences from working in and for the Danish Folkeskole combined with my master degree in mathematics and pedagogy.

I finished my pre-service education as a teacher of mathematics and arts in 1975. For the following ten years, I worked as a teacher in three different public schools. I had all kinds of classes in all grades, as is the norm for teachers in the Danish school system.

For the next eleven years (1985-1996), I worked for the government at the National Innovative Centre for General Education (in Danish: Statens Pædagogiske Forsøgscenter or just SPF). The centre is in Copenhagen and consists of a school and a Youth Town. The School teaches only pupils from grades 8-10, about 14 to 17 years old. One of the purposes for the Centre is to pilot and pioneer work which would inform later Educational Acts (The current government decided to close the SPF in July 2007). I was hired to develop mathematics teaching for those grades, and to write articles and reports based on the ideas and results obtained from the experiments. During my time there, I taught different subjects, but my main objective was to design experiments that could help the students to a better understanding of mathematics. Meanwhile, I graduated with a master degree in mathematics and pedagogy in 2001 from the Danish University of Education (DPU). Along the way, certain seminal experiences or situations influenced my decisions and beliefs in my work as a pedagogical consultant:

1. In Denmark, the same teacher usually teaches the same pupils all the way from grade 1 to 10 and not only in the subjects that the teachers were particularly trained for. Therefore I taught not only mathematics and arts, but also other subjects like Danish, biology, music and geography. My experiences during those years were very varied, but every so often, I wondered if my students were taught well enough, and I often felt that my training was not adequate, particularly in the subjects I was not trained in.

In many schools in Denmark, a given class only has a few teachers. The rationale is that it is better for students to be taught by a few teachers they know well than having many teachers, each with subject-specific expertise.

2. While I was at the SPF, I worked closely together with other teachers who were commissioned in the same way as I was, but in other subjects. The tools we used in our collaboration were, among others, ‘project work’ and ‘aesthetical learning processes’ (ALP). ALP is a term adopted from a theoretical framework in arts education, (Hohr and Pedersen, 1996). I should make it clear that the word ‘aesthetic’ when used here, does not connote beauty. Rather, it is used in the original Greek meaning of ‘aisdissertation’, meaning ‘knowledge that comes through the senses’. The

results we found as we worked with these tools have influenced my preparation of any teaching ever since.

2.2 The initial in-service course

'Preparing for Mathematics Exams' is the title of the in-service course in focus of this dissertation, which is a direct attempt to implement the ministerial orders for the examinations after the ninth and tenth grades.

The description of the course in the 2002 catalogue, reads as follows:

> *Goal: To qualify mathematics teachers, who are required to run an oral examination in mathematics based on the departmental order and to create connection between the exam and the day-to-day teaching.*
> *Content: We will discuss how the oral examination could be organized, and how the day-to-day teaching and the exam influence each other. The application of IT will be a part of the course. Based on this, the individual teacher will design a course for her or his own class and run it in the period between the first and last part of the in-service course. In the last part of the in-service course the work is based on analyzing and assessing of the courses the teachers ran in their own classes.*
> *Target group: Teachers who want to prepare for organising and running the oral examination related participatory to the day-to-day teaching.*
> *Price: DKK 3.170 (my translation)*

At the particular course in 2002, 27 teachers enrolled, consisting of eleven women and sixteen men with different kind of basic education in mathematics. The preparation of this course was based on the structure of the first course, the prototype developed in 1996. When I speak about one of the courses, I shall mention the year to prevent confusion between the prototype and one of the later year's courses, e.g. C-02.

Before my Ph.D. study, the in-service course was offered for the first time in the autumn of 1996 by the Royal Danish School of Educational Studies, and it has been repeated with few changes at least once a year every since. From 2000 onwards it was offered by the institution "Centre for Further Education, Copenhagen & North Zealand" (CVU, Copenhagen & North Zealand). The in-service course was designed and run by me all the way through; sometimes I invited a guest teacher-educator to do a lecture for the class.

The participating teachers came from different schools and enrolled individually based on the written description in the catalogue. The current course was aimed at teachers who need support to conduct the oral examination at the end of lower secondary school and to understand how the exam influences the day-to-day teaching. At the first course in 1996, two hundred teachers enrolled, whom I organized in three courses, one with a hundred participants and two with fifty. The average number of teachers enrolled in the courses in the last five years, has been about thirty. The fact that so many teachers participated in the beginning shows that teachers feel a need for in-service courses when new ministerial orders are introduced and it seemed that even after a number of years, there were still teachers each year who found the course relevant.

The in-service course in 2002 lasted for forty-eight working hours, of which thirty hours were placed in one week, and the remaining eighteen hours were taught over three days two and half months later; for this particular course I was also asked by my employer to spend some time on IT.

The prototype of the in-service course consisted of three parts: the first fifteen hours were devoted to the interpretation of the ministerial order for the oral examinations, discussion about how to formulate open practical problems, practical exercises, where teacher-students created problems and evaluated them as well. The next fifteen hours were set aside for planning a course to be taught in the individual classes during a two months period. The third part comprised two days two and a half months later, where we reflected on the individual teacher's experiences from the teaching 'in-between'; here communication and working with open practical problems were the primary focus. In this part we made use of the teachers' logs, which was handed in before the last round. The instruction for how to write these logs was given by me. The instructions call for teachers to outline: Communication with and among the pupils and examples of pupils' problem solving strategies; what the teachers experienced as a success and why; what was difficult and how could it be made easier. The in-service course was an interaction between input from me, discussions, workshop-like activities and group work; both the process and the results were made objects of reflection. The overall pedagogical approach was based on the belief that the student-teacher should be active: 'Learning by doing'. It means that the course consists of small workshops with situations that provide different sorts of input to the teachers. The activities in 2002 were developed from my own experiences as a teacher and teacher-educator, combined with research ideas. The teachers were not required to read the research; I just mentioned the relevant theoretical frameworks. The 'owners' of the theoretical ideas used in the different modules are listed along with the activities.

2.2.1 Details from 'Course 2002'

The following description contains some of the details from the in-service course held in 2002. It is a combination of the preparatory considerations and what was realised; the difference between my conjectures and reality will be explained and discussed later. The course ran each day six hours from 9 am to 3 pm and contained approximately three periods of 1½-2 hours each, two before lunch and one after. Days six, seven and eight are the third sequence of the course:

Day 1:

Period I: Introduction and discussion of the ministerial order of the oral exam. Materials were overhead transparencies of the ministerial order and some questions I had prepared to start the discussions. I also gave an introduction to the difference between open and closed problems (Skovsmose, 2000; Becker & Selter, 1996; Pehkonen, 1997) and to mathematical modelling (Blomhøj & Jensen, 2002, Christiansen et. al., 1997; Ejersbo, 2000).

Period II: Discussion of 'what is a good problem?' The teachers discussed their own tasks in groups before discussing them in plenary. Materials used for the questions were tasks from textbooks or practical problems used in oral examinations and brought to the course by the teachers.

Period III: Teams were set up based on a shared interest in the topic of each task. The assignment was to formulate a task with an open practical problem, which could be used at an oral exam.

Day 2:

Period I + II: Use of IT at the oral examination. This was a lecture given by Professor A. C. Malmberg, who was a colleague of mine that time. I attended this lecture. His presentation consisted of the history of and introduction to the INFA research project, A research project about IT software for mathematics, developed at the Royal Danish School of Education Studies.

Period III: Group work with production of open practical problems for an oral examination. The end product was to be three pieces of papers, one for the pupils, another with an explanation for other teachers (or the external examiner), and the third with a list of necessary materials needed to solve the problem.

Day 3:

Period I**:** A ‘role-play’ (see fig. 3) where some of the teachers were ‘pupils’ at an make-believe exam, while others were ‘teachers’ or ‘external examiners’. The tasks applied were those that the teachers just made themselves. The purposes of this ‘role-play’ were to evaluate the tasks, to practice how to ask questions, and to determine how to evaluate the exam presentations. Reflection was done in the groups after this session.

Period II: After finishing the role-play, the teachers met in the ‘task groups’ to make a final version of the task incorporating feedback from the evaluation. All the tasks were then distributed to all the participants.

Period III: Workshops, including a presentation of the theoretical framework ‘didactical contract’ (Brousseau, 1997) and different tasks, both meant as an inspiration for the preparation of participants own mathematics teaching. The tasks represented different approaches to talking mathematics (Clarke, 1996).

Figure 3: A ‘role-play’, where some of the teachers are ‘pupils’ while others are ‘teachers’ or ‘external examiners’ at an ‘oral exam’.

Day 4:

Period I: A short introduction about how to use subject writing in mathematics (Parr and Falck-Ytter, 1994; Abildgaard and Mogensen, 1999).

Period II: The teachers prepared a school course with their groups.

Period III: What is a cognitive conflict? (Holbech and Thomsen, 2000; Adey & Shayer, 1994) After a short introduction, the teachers continued their preparatory work.

Day 5:

Period I: A short task about communication and understanding: The teachers worked in pairs where they took turns to present a pattern, each of them made of coloured pieces on the table, and explain it to the other.

Period II: The planning work continued until lunch, at which point all the products were printed and handed out to each participant.

Period III: Short presentation of different subjects, such as the writing exam tasks, the KOM (For more information about this project see: Niss, 2003) project (Niss and Jensen, 2002), and the PISA project - a fast flow is necessary Friday afternoon to make the teachers stay at the course. We finished this part with an evaluation of the first five days, both oral and written. The evaluation questionnaire was designed by me (see appendix I).

This 5-day course was followed by a ten week 'break', during which the teachers ran their planned courses with their own pupils. During this period the teachers kept a kind of log and collected their pupils' work. Selected parts of these logs were sent to me before the last three days. The teachers were encouraged to stay in contact with each other in their working group, but I was not directly involved in this networking.

Day 6:

Period I: The old working groups met and prepared a presentation of their work in the different classes.

Period II: Presentation and discussion.

Period III: Examples of IT used in the oral exam presented by a guest teacher educator, E. Ladefoged.

Day 7:

Period I + II: IT continues with the same visiting teacher.

Period III: Workshop about reflection on action: how to make improved action more reflected*.

Day 8:

Period I + II: Workshop with tasks and further discussion about how to reflect on action.

Period III: Debate: When is communication every-day talk and when is it mathematical talk, and what is the difference? Evaluation of the entire course, the questionnaire was an official one designed by the institution (see appendix II). We finished the in-service course with a visit to a text book exhibition, where we took part in a presentation of different mathematics text-books.

The overall plan and strategy was based on the pedagogical notion that the participating teachers should be active in their own learning process: Learning by doing as Dewey expressed it; this principle will be explained in details in a later chapter.

*The reflection took place in timeslots set aside for some task, but not directly taught. The teacher-students were asked to reflect in their groups, but no tools for reflection were given. My implicit assumption was that the teachers would know how to reflect as long as I gave them some time to do so it and some tasks to reflect on, but I found I was wrong with respect to the need for teaching about reflection. In this study, I found that it is not a given thing that mathematics teachers know how to reflect professionally on mathematics teaching. Hence it is necessary to teach this particular topic very carefully; more on this later.

2.3 Learning Lab Denmark

I began my Ph.D. at Learning Lab Denmark (LLD) in 2002. LLD is physically placed in the same buildings as the Danish University of Education (DPU) and was integrated into DPU in January 2005. LLD was established in 2000 with a mandate to experiment in response to changes in educational research. From the beginning, the overall methodological approach has been that of action research, characterised by demands for research from outside of academia. The LLD projects are often externally evaluated and chosen by users and markets, not to be mistaken for 'mass market', but rather the many different recipients, target groups and stakeholders of the artefacts in questions. Thus, in order to ensure successful intervention in educational practice, implementation should be considered equal in importance to development of theory and artefacts.

During the first three years, I only studied part time, while I continued my work as a pedagogical consultant alongside my Ph.D. project. The present study is based on one of the courses I ran in this period. When I began the study, I was inspired by the participating teachers' reaction to the course. The post-course evaluations I received from the teachers were often very good. In 1997, I did a follow-up study of the 1996 groups, to see how the teachers used the tools. Of the two hundred teachers who participated this study (Ejersbo, 2001) some 48% responded; 96% of those responded positively to questions about the course. Even older 'student-teachers' expressed satisfaction with the in-service course and told me how it had changed their teaching. I wondered how exactly their teaching practice had changed because of the course, and gradually this became the reason for my Ph.D.

The kind of in-service courses studied here, is a typical in-service course used to implement new curriculum initiatives in Denmark. Yet little is known about how it influences classroom practice. In the beginning, my study and inquiries included analysis and interpretation of the classroom observations where the teachers taught mathematics by means of open practical problems. Thus, the study showed me the in-service course from new perspectives, and therefore the study also includes a re-design of the in-service course, testing of the new design, documentation, and discussion.

My study fits into the LLD-framework through the design of the in-service course that could be seen as a commodity on the market. If nobody signs up, there will be no in-service course. The participating teachers enrol voluntarily, and therefore they are customers, so to speak. The course must strike a balance between teachers' needs and the Educational Act's requirements – and preferably also find a fertile compromise between them.

The research unit Science, Technology and Learning (STL), which I am a part of at LLD, is specialized in design and production of learning games. Yet, everyone in the STL-group tries to combine design-based research with the original problem-based action research.

2.4 Goal and research questions

The goal of this study is to use empirical investigations and theoretical discussions to examine an in-service course for mathematics teachers; a course on using open practical problems in mathematics. The research consists of a pre-study focusing on the participating teachers and the benefit they gained from the in-service course which was run in 2002 (C-02).

In the pre-study, a further inspection of C-02 was made in order to ascertain the following: To what extent and in what way did the participating teachers adopt teaching with open practical problems in their classes, and what kind of problems did they have, seen in the light of how those problems were addressed in C- 02? A procedure for redesigning my course was drawn up based on the results.

My research question concerns different aspects of the redesign of the in-service course, and brings together two main themes of my research. One theme is the successive redesign and teaching of an in-service course; the second is the didactical transposition of theoretical ideas from mathematics education research literature into the practice of the in-service course, hereafter referred to as meta-didactical transposition.

The Research Question is:

To what extent and in what ways can a meta-didactical transposition be incorporated into the successive stages of a redesign of the in-service course, and how effective is this redesign, measured by the participating teachers' reactions on the course?

As with the pre-study, in-service education is a part of the life-long-learning idea with the aim to help a profession meet new demands; in this case new requirements for an oral exam, and also to help teachers learn to design open practical problems and get to know how to teach mathematics through them. My investigation revolves around the teachers' beliefs concerning using open problems in mathematics and how they maybe used it in their own teaching. Furthermore, I wanted to examine to what extent the teachers grasped the aims with the course, since my earlier survey (1998) indicated that teachers changed their teaching on basis of the course. The pupils' reactions and enthusiasm working with open practical problem are of interest as well, obviously. As mentioned above, I sent out three tasks together with a questionnaire before the course, to determine how familiar pupils were with solving open practical problems and whether they liked it. There is not just one way to use open practical problems in an effective manner, but there are ways that work better than others. I wanted to see for myself whether and how the teachers were able to use open tasks after the course, and through this evaluation see how effective the course was. I used interviews and videotaped classroom observations to answer the question concerning the pre-study.

The redesign itself was based on certain guiding principles for teaching, which I developed based on my analysis of the results of the pre-study. The focus is on the teacher educator's redesign of modules for communication and reflection prepared through following the guiding principles for teaching, as well as on the participating teachers' feedback to this work. Comparing my expectations about the learning goals of the course with the reality in the school classroom, I wondered why the teachers used only some of the elements from the course but not others. Investigating C-02, I realised that the instruction design concerning communication and reflection could be significantly improved through a redesign. The redesigned instructions and the teachers' reactions comprise the data I used to answer my research question. In this part of the study, I used design research to collect and analyse data from in-service courses 2004,'05 and '06, where the redesigned instructions were realised. The data are once again teachers' logs along with video- and audiotapes from the courses.

Investigating the process behind the re-design, we can look for an explanation of the researcher's process for transposing theoretical ideas from mathematics education research literature into the practice of an in-service course. This process, I call 'meta-didactical transposition' by inspiration from Chevallard's concept 'didactical transposition' (Chevallard, 1985), which denotes the process where mathematics as a scientific subject is transformed to be taught as a 'mathematics-in-school' subject.

The teaching at the Royal Danish School of Educational Studies was meant to be research-based, but it was difficult to determine just what this implies and it never became clear how it should be realised. In the 1990s, when new teacher-educators were employed at the Royal Danish School of Educational Studies, they were not mostly researchers. This new group of teacher educators was a success from the point of view of the teachers and the market. The methods and materials were often developed during their own school experiences, and transformed into in-service teaching. It was, so to speak based more on experiences than on research. What I wanted to investigate was how to transpose research ideas into in-service education, taking both the content and the teaching methods into account; and to develop a new theoretical framework for in-service education.

2.5 Research focus

The design of the first in-service course took place before this study began. Figure 4 shows the courses of the in-service courses during the years.

1996	…	2002	2003	2004	2005	2006

Figure 4: The first in-service course was designed and conducted in 1996 and at least once a year ever since. The first in-service course in the Ph.D. study was run in 2002 and slightly revised in 2003. The re-design was prepared and tried out in 2004 and revised again in 2005 and 2006.

The classroom observations, of the subsequent teaching in school classes together with the interpretation of the collected data, comprise the basis for the recurring redesign of the in-service course.

In my research, I noticed a gap between my expectations for the in-service course and what I observed in the classroom. Occupied by providing what I would call good in-service education, I became aware of the different ways in which good in-service education could be interpreted. From a market point of view, the in-service course was a success year after year and it was obvious to ask why I would want to fix it if 'it ain't broken'. The teachers were satisfied with my preparation and the way I taught the course; and they were still satisfied when they were back in their own schools. Therefore, my initial reaction was merely to observe what took place in the classrooms. Yet, the analysis and subsequent interpretation of the data revealed that something was not going as expected. My own implicit assumptions about what the teachers would take with them from the in-service course were not in line with what I observed. I became aware that it is not always possible to meet both the participating teachers' expectations and to help them learn how to generate competencies for their professional duties, e.g. how to reflect on their own professional behaviour. Reflection often calls for leads to changes at a personal level, which may be a cause for anxiety (Fullan, 1985; here (Pinar et al., 1995), p. 702). If this anxiety develops to an unpleasant level, teachers may stay away from the in-service course. It is an advantage for everybody if the participating teachers are happy with the course, but perhaps it is not always possible or sufficient. The key issue here is that, at the personal level, the changes must comprise a professional development of competencies and not be construed as an attack on the personality. Furthermore, the tools should be generative for the teachers. Teaching is a cultural activity, but it is the teachers who are criticized if the official curriculum goals are not achieved. I concur with Stigler and Hiebert (Stiegler and Hiebert, 1999), when they say that focus should be on the methods used in the classroom and how to develop them, rather than on the teacher's personality. In this study, I examine teaching methods, how to develop them, and the range of reactions generated when they are realised so, this study is not about teacher development, or about how pupils learn mathematics through open problem solving.

In-service education is subject to market conditions, but it is not always just a pleasure to learn new skills. In this case the question 'how to communicate and ask powerful questions that will encourage the pupils to work with and learn mathematics' is important, but difficult to answer, and is in itself an open question. It is in many cases a question about awareness (Gattegno, 1970) and hence reflection as well.

My own process has engendered a new and improved understanding of my own implicit assumptions about teachers' knowledge and competencies. This new understanding influenced the way I made up some rules, which I used for preparing and running the redesigned in-service courses; it is therefore relevant to describe my own process and from that generate meaning as well.

3. Methodology

3.1 Overview

My study is a 'one-person' research project and is not a part of any other project or collaboration. I planned and organised the research, collected all the data and transcribed and interpret the data myself; of course, I have received help from my supervisors. The study is a part of the field called the didactic of mathematics, which Niss (Niss, 1999) p. 5) defines according to the four components: Subject, Endeavour, Approaches and Activities. In my study these components are realised as follows:

The subject is a recurring in-service course for mathematics teachers in lower secondary school, aimed to teach teachers how to teach through open practical problems in mathematics. My aim is to identify what the teachers learn from the in-service course, and to understand why some tools are more successful than others.

The endeavour is to look for any causality between what happens on the in-service course and in the school classes after the teacher has participated on the course.

The approaches in this study are methodologies that help to enlighten what makes it difficult to teach mathematics through open practical problems in mathematics, research concepts to find out how in-service education could be organized taking the difficulties into account, and to investigate how it is possible through a qualitative methodology to answers complex questions without strong scientific validity.

The activities comprise two periods of empirical research, with classroom observations and in-service classroom observations, and for one part theoretical work, involving literature studies and new theoretical ideas.

The in-service course was developed in 1996, and the general design for this first course still provides the framework for the subsequent courses. This means that both the prototype and the subsequent courses are potentially relevant for my study. The prototype is norm-related, while analysis of the realisation of the particular courses is descriptive, in the sense that what is described is what actually transpired. The same difference can be observed in the school classes: What are the teacher's views or belief (norms) compared with how the teaching is realised in the classroom (descriptive). What was the teachers' (normative) outcome from the course and how was it taught back in the classes (descriptive) related to the teachers' own description of their beliefs on planning and teaching (their norms) and the teacher educator's expectations to, and normative goals for the course?

The teaching, learning and outcome can be viewed from both a descriptive and normative perspective. Furthermore, it can be viewed from the teacher educator's position or the researcher's position. The different points of view are likely to provide different opinion about the descriptive and normative perspectives. The following figure by Niss (1999) show different levels of mathematical didactics.

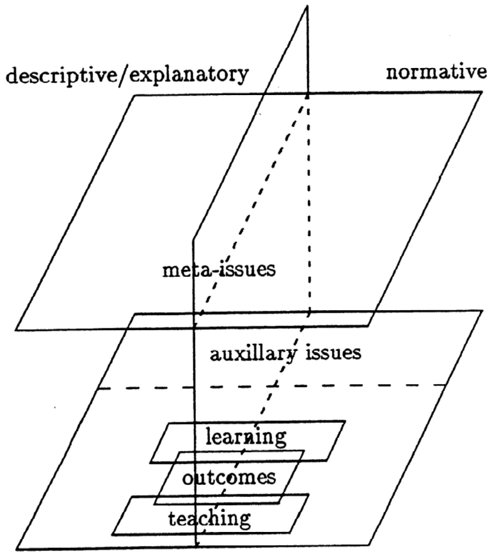


Figure 5: Survey map, after Niss (1999)

The 'ground floor' consists of teaching and learning intersected by the outcome and auxiliary issues. The upper floor is the 'meta level', and contains meta-issues. Both floors are defined by their different views: the normative and the descriptive views.

This notion of the differentiation between normative and descriptive is important in my investigation, where the teachers' or the teacher educator's beliefs belong to the normative part when they are developed, while they can be described when they exist together with the realised practices that are descriptive and not always in accordance with the beliefs. The different levels play a role in my work as well. If the ground floor level concerns the teaching/learning issues in the school, we can use the figure as an analogy and say that the teaching/learning issues at the in-service course, which is about teaching/learning in the school, is a meta-issue and belongs to the upper floor. If we consider at teaching/learning issues at the in-service course and discuss the beliefs and meta-issues about this kind of teaching, then we have the teaching/learning at the ground floor and the discussion and the meta-didactical transposition at the upper floor.

The auxiliary issues are related to the approaches and revolve in this study around learning processes and teaching in learning processes. It means that pedagogy, psychology, and sociology are involved, helping me to answer my research questions.

While Niss is a researcher who seeks answers to questions in the didactics of mathematics, Wittmann (Wittmann, 1998) is a designer who describes mathematics education as a 'design science'. He explains the core of didactics of mathematics and the areas related to mathematics education as aimed at an interdisciplinary, integrative view of different aspects and at constructive development; whereby the ingenuity of mathematics educators is of crucial importance (p. 89). With the following figure, Wittmann shows how he understands the relationship between the core of didactics of mathematics and related areas, which is a help for the designer of mathematics education.

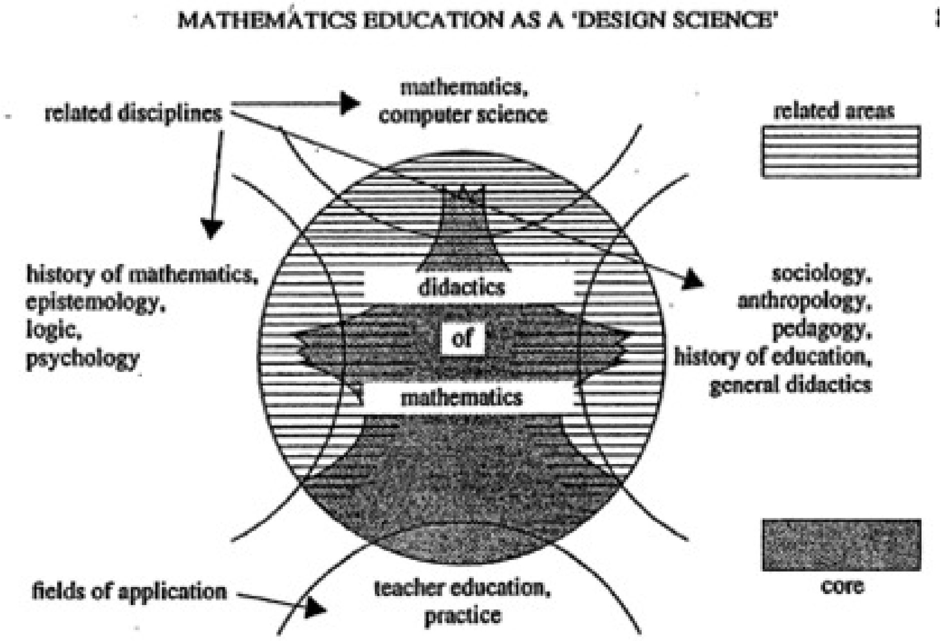


Figure 6: The core and the areas related to mathematics education, their links to the related disciplines and the fields of application, after Wittmann (1998)

Wittmann used Freudenthal (Freudenthal, 1991) to explain:

> *However, the division between the core and the related areas does not imply that the core is restricted to practical applications since the related areas have to develop the necessary theory. In fact, building theories or theoretical frameworks related to the design and empirical investigation of teaching is an essential component of work in the core. (ibid, p.89)*

With this view, he expresses that the core, the related areas and a ‘lively interaction’ between them, represent the full picture of mathematics education and the responsibility of mathematics educators.

In my study, I have used design research to collect and interpret some of my data. In Wittmann’s case, he uses the expression design for creating mathematics education for mathematics-in-school, while I used design research for the way I structured the second part of the study, and for the re-designing of the courses C-04 to C-06.

Wittmann claims that the teaching unit has not been sufficiently accepted as research so far, and explains this with the researchers’ resistance to put themselves on a par with teachers. Yet, it is crucial to change this tradition, he says, because “the design of mathematics teaching units is a most difficult task that must be carried out by the experts in the field”. Wittmann recommends research in mathematics education in the form of sets of carefully designed and empirically studied teaching units that are based on fundamental theoretical principles. In the French discussion of ‘didatictique des mathématiques’, ‘didactical engineering’ is a related terminology meaning to engineer examples of good teaching (Artigue, 1994).

I do not uphold any specific ideals for how to teach in my study, but rather, when developing my own in-service teaching to be more effective, I tried to find out what drives and qualifies the design of the instruction on the course so as this become a catalyst for the teachers’ professional development. My goal was to develop a model for applying research in both the content and the methods of teaching. Inspired by Strauss and Corbin (Strauss and Corbin, 1998), I looked for elements that could be evolved. This means that I did not stop with description and interpretation, but I designed and constructed solutions, realised them and investigated the outcome as well. I knew that there usually is a gap between intentions and implementation, so what I was looking for was the gap, and for ways to narrow it. Furthermore, I wanted to contribute to theory and practice, with a focus on the relationship between the theoretical mathematical ideas developed by didactical researchers and how those ideas could be transposed into practice and realised in the teacher-students' teaching. I hoped in this way that the learning phenomenon was more in focus than the methodology, and that the findings, strengthened by the hypothesis or theoretical frameworks I attempted to develop, could be useful for both practitioners and researchers.

My role during the in-service courses was both to be the teacher educator and a researcher. Observing and exploring my own practice at the in-service courses made me an 'insider researcher', while observing the school classrooms made me an 'outsider researcher', who explored the practice of others. Jaworski (Jaworski, 2004) attributes the expression 'insiders' and 'outsiders' to Bassey (Bassey, 1995) and defines them as:

> *Insiders and outsiders are terms used to refer to researchers who explore their own practices (insiders) and researchers who explore the practices of others (outsiders) (…) insiders may be more interested in justifying research conclusion in terms of warranted practice, whereas outsiders may seek warrants related to knowledge and theory. (Jaworski, p.3)*

The research topic in this study was both the teachers' practice and my own in-service practice. I studied the teachers' practices to collect data for evaluating my own in-service teaching. So, when I observed the teachers' teaching, I had a double role, because I had been their teacher. As an insider, I was interested in the way my own practice could be developed based on reflection-on-action (Schön, 1983) (the term reflection will be explained in further details later) and on the outcome I saw in the teachers' practice. In other words, the interpretation of the outside research provided knowledge back to the redesign of the in-service course. I call the product a combination of 'inside and outside' research.

The double role created some of the same complications as discussed by Wong and Wilson (Wong, 1995). There was, from time to time, a conflict of interests. To be a teacher is to impartially interfere with the pupils – in this case the teachers – while to be a researcher is to observe what happens. Others have studied their own practice and made it researchable. I have been inspired by their methods in my own work: Lampert (Lampert, 1990,Lampert, 2001) who taught a school class for a year and described her experiences from that year, Ball (Ball, 1988) (Ball and Bass, 2000,Ball and Bass, 2003) who taught as well in primary schools and in the same time was a researcher, and Hviid (Hviid, 2003) who described her experiences from teaching psychology at the university. I made a firm decision that when I was teaching at the in-service course I was solely their teacher educator, and when I interviewed or interpreted the data I was solely a researcher. I must admit that it was not always easy to uphold this distinction. I did the videotaping at the in-service courses with the help of some of the teachers, and for the tape recordings, if the tape ran out in the middle of a situation or the recorder was placed badly, I accepted it without fixing it, because of my priorities.

I had different issues to investigate and different methods that I used to collect and interpret data for my three research questions. Therefore, each part and the methods used will be described separately.

3.2 The pre-study

In the pre-study, a further inspection of C-02 was made: while examining to what extent and in what ways some of the participating teachers adopted teaching with open practical problems in their classes, I looked for the problems the teachers had and compared them with how those problems were addressed in C- 02. Based on these data, a procedure for a redesign of the course was made.

My investigation pertaining to this question began before I knew who or how many teachers were enrolled in the in-service course. I began the preparation of my study in the spring of 2002, and I decided to use the course 2002 as the object of my investigation. In May, I had all the names; 27 teachers enrolled in that particular in-service course, which was offered by the CVU and carried out at the DPU.

My intention was to involve as many of the enrolled teachers as possible. In May 2002, my first move was to write all of them a letter and explain the research situation: that the course was a part of my project. I asked them to get back to me via email with their acceptance or not to be a part of my research, but only three actually wrote me back. I phoned and asked all of them personally to be a part of the research project, to which they all expressed a positive interest, and then I received three more written replies. I phoned the rest once more and asked them to be a part of the research, and I received two more replies for a total of eight. I was somewhat dismayed that it turned out to be so difficult to get the teachers to participate in the research; on the other hand, I see now that the potential benefit of their participating in the research was small and uncalled on for their part, and to make matters worse, participation would take time and maybe give them more work. During the in-service course, more of the teachers expressed interest in the research, but I decided only to use the first volunteers for interviews and tasks because their pupils had solved tasks and filled in questionnaires; but in a way they all participated in the research just by being on the course.

The eight volunteers received three tasks along with a questionnaire for their pupils. The tasks and the questionnaires were formulated by me and tested in a mathematics eighth grade in a regular public school; the teacher had participated in a similar course in 2000. I received some suggestions from that class, and I subsequently reformulated the tasks to be easier to understand. From the eight volunteers I received 144 solved tasks and completed questionnaires.

Of these eight teachers, I interviewed five. The reasons for not taking all eight were partly practical, since it proved to be very difficult to make an appointment about a visit, and partly scholarly as I found the five teachers to be representative for the group. The five teachers, three women and two men, came from different places and schools. Two of them worked in public schools in Copenhagen, while the other three worked in public schools in smaller towns outside Copenhagen. The Copenhagen schools had some 600 pupils each, while two of the suburban schools only had 400 pupils; the last one had more than 800 pupils. Four of the teachers had less than six years of experiences, but were all specialised as mathematics teachers. The fifth teacher had 20 years teaching experience, but was not trained as a mathematics teacher for lower secondary classes.

3.2.1 Collecting data

For the interviews, I prepared a semi-structured interview-schedule predominantly based on Kvale (Kvale, 2001) and Patton (Patton, 2001). From another research project, I had some experience with group interviews, and I prepared for the individual interviews by practicing on a teacher (M.

Skånstrøm), who is a former colleague and who is not a part of the research project. This interview helped me to make more specific questions. Instead of asking, "What do you believe makes a good mathematics lesson?" I asked: "What happened in that mathematics lesson, where you left with the feeling that it was a good lesson?" All the interviews were taped, including the first one, and transcribed in two ways. One version for me; here I wrote what was said and the pauses, the laughs etc. This version was written the same day as the interviews were made. The other version, I sent to the teachers for approval; in this version I deleted my own comments and the transcription codes. The interviews took place at the teachers' schools about two months before the course and were taped on a recorder. The open structure of the interviews was a help, both for me and for the teachers. With these interviews and with an investigation, in which all of the participating teachers wrote down their expectation in the beginning of the course and commented on those expectations in the end, I obtained a documented and complex picture of the teachers' expectations of the in-service course. Using qualitative methods for coding interview data, I categorised the data (Glesne, 1999) in five categories: Expectation, characteristics of a good mathematics lesson, consciously inspired influence, colleague cooperation, and obstacles.

I collated material used for the teaching, the tasks produced, and logs from the teachers, my own log, pictures and evaluation sheets from all the participating teachers from the course 2002. I used all those raw data as documentation of my own investigation and for final evaluation of the in-service course.

Of the five teachers, whom I chose to interview in the first round, one became pregnant and therefore she stopped taking part in the inquiry. Of the remaining four, one stood out from the others; she had twenty years of experience as a teacher but no specialisation in mathematics or teaching experiences in mathematics. I therefore chose only to interview her in the second round, without visiting her class. The last three were two men and one woman. They had two to five years of experience and were all specialised as mathematics teachers. I visited them and their classes with a video camera to observe their classroom and a tape recorder for a subsequent interview – after I had obtained permission from the pupils' parents. These three classroom observations are analysed as case studies, where I focus on the teacher's difficulties when teaching mathematics through open problem solving.

3.2.2 Interpreting the data

Inspired by the μ-Group's (Pirie et al., 2001), Powel et al. (Powell et al., 2003) and Jordan-Henderson (Jordan and Henderson, 1995), I watched the raw videos of my shooting in the school classes, a couple of times before I put in headlines for each five minute-sequence, and decided how to categorize. Certainly, I had some categories in mind before I recorded the video, but as recommended in Strauss and Corbin (1998), I let the data talk to me too. One of the classes was a ninth grade and two were eighth grades, coincidentally the same as in the TIMMS Videotape Classroom Study (Stiegler and Hiebert, 1999). The way the TIMMS videotapes were studied was an inspiration to me, both for procedure and for the comparison. One of my study tours was in Norway at 'Høgskolen in Agder' (HIA) where I took part in a seminar with Professor Roger Säljö. At the seminar, I learned to interpret videotapes in a 'qualitative' way:

1. We looked at a selected five minute-segments, without making any comments;
2. We received the transcription of the five minutes and watched it again, still without making any comments;

3. The third time we were allowed to comment and the videotape was stopped at once, when someone spoke. We could make any comment we wanted. Our comments were discussed immediately.

I used this method among others for the interpretation of my video-taped data.

When I made my transcriptions, I used very simple coding scheme based on what I needed the transcription for. The classroom observations focused the teacher's communication with the whole class, groups of pupils or individual pupils. I wanted to determine how the teacher listened and responded, and the level of awareness and reflection in and on practice. In the subsequent interview with the teacher, I asked for the teacher's own feelings and experiences with the lesson just finished, and related the answers to some of the comments in the first interview. The semi-open structure for this interview was designed specifically for the individual teacher.

3.2.3 Ethnographical investigation

The classroom observations could be viewed as an ethnographical investigation into a community setting with which I am very familiar. I have been a teacher for many years, and I know the kinds of classrooms I visited very well. Therefore, I may see other things than if I was a newcomer in the area. At the same time, I could be 'blind' concerning 'trivial' things. Calling my investigation 'ethnographic' is based on, among others, Forsythe (Forsythe, 1998):

Ethnographic methods include participant observation, formal and informal interviewing, and analysis of documentary material. These methods are flexible and designed to be adapted to a real-world situation. In addition to producing detailed, understanding of real-world social processes, they also provide insight into concepts and premises that underlie what people do, but of which they are often unaware. (Ibid, p. 405)

The way ethnographers usually refer to forms of social research is characterized by Atkinson & Hammersley (Atkinson and Hammersley, 1998):

- *a strong emphasis on exploring the nature of particular social phenomena, rather than setting out to test hypotheses about them*
- *a tendency to work primarily with 'unstructured' data, that is, data that have not been coded at the point of data collection in terms of a closed set of analytic categories*
- *investigation of a small number of cases, perhaps just one case, in details*
- *analysis of data that involves explicit interpretation of the meaning and the functions of human actions, the product of which mainly takes the form of verbal descriptions and explanations, with quantification and statistical analysis playing a subordinate role at most. (Ibid, p. 110)*

The ethnographical classroom observations held gave rise to more diverse information than what I could have discovered through interviews alone. I realised the truth of what Kegan (1980) said:

(R)esearchers and practitioners do not learn about person's meaning-making system by asking the person to explain it, but by observing the way the system actually works (after Simon et. al., 2000)

There were more traps, when studying the classroom, interviewing the teacher and interpreting the data. The interpretation was a journey, where I started with a kind of emotional 'what I would have done different' feeling; the old teacher in me was not neutral. My emotions were a parameter in my interpretation for a long time, and it was difficult to differentiate the data for the purpose that drove this dissertation. I felt the paradox that is inherent in this kind of research (Simon, 2001): What I noticed and paid attention to was how I understood teaching and learning of open practical problems in mathematics, even though I wanted to make sense of the ideas and actions of the teachers' understanding. This important process became a part of the study.

3.3 My Research Question

My research question concerns different aspects of the particular type of the in-service course, which I designed and ran and it brings together the two main themes of my research: the successive redesign and teaching of an in-service course and the didactical transposition of theoretical ideas from mathematics education research literature into practice of the in-service course, called meta-didactical transposition.

The Research Question is:

To what extent and in what way can a meta-didactical transposition be incorporated into the successive stages of a redesign of the in-service course, and how effective is this redesign, measured by the participating teachers' reactions on the course?

The process pertaining to this question began with an investigation of the instructions given at C-02 that should help the teachers develop communication and reflection in their own teaching. In doing this, I found that the instruction itself lacked efficiency, and that there was a need for a redesigned version.

The data for the first part of my research question was predominantly empirical. To develop new instructions, I had to do some research in the literature. I searched for research concepts related to mathematical communication, including listening and questioning, and reflection. In the spring of 2003, I participated the conference CERME 3 (Third Conference of the European Society for Research in Mathematics Education), held in Bellaria, Italy. In this conference, I was in Professor B. Jaworsky's and Professor K. Krainer's group, where I met Professor H. Steinbring. Because I showed an interest in his work, he mailed me both his books and his articles after the conference. Steinbring's theoretical framework about the 'Epistemological Triangle' became one of the concepts used in the redesign. At the same conference I met Professor U. Leron and became familiar with some of his work, the 'Virtual Monologue', written in an article with Professor O. Hazzan. The Virtual Monologue is also implemented in the redesign. I was familiar with some of the work of Professor A. Sfard and I found the 'Flowchart diagrams' that she had developed with Professor C. Kieran, relevant as a means of instruction. For the last theoretical concept, I chose to transpose Professor P. Cobb and Professor E. Yackel's work 'Sociomathematical Norms' for the redesigned instructions.

3.3.1 Design research

I used design research for developing and investigating the outcome of the redesign. I focused on the design of teaching units with the intention of generalising those units to guide the design process. The benefit of using design experiments is that we can simultaneously develop theoretical frameworks and in that way improve practice. In this study where I was both the researcher and the teacher educator, I looked for improvements from different positions. As teacher educator my intentions were that the 'teachers' (means from now the teachers on the in-service course) would learn new skills that improved their teaching, but as a researcher I just wanted to observe without changing. The combination that the research can be used to improve the artefact – in this case the in-service course – and in the same time develop theoretical issues is one of the main ideas using design research.

Kelly (Kelly, 2003) describes design research as

> *An emerging research dialect (…) attempts to support arguments constructed around the results of active innovation and intervention in classrooms. The operative grammar, which draws upon models from*

design and engineering, is generative and transformative. It is directed primarily at understanding learning and teaching processes when the researcher is active as an educator.

My study seemed obvious to take advantage of design research. I tried to understand the learning and teaching processes that took place on the in-service course, where I observed the reaction and the outcome reflected by the teachers acting and descriptions.
Barab (Barak and Squire, 2004) mentions that design research is a

(…) series of approaches, with the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching in naturalistic settings

Kelly (Kelly, 2005) quotes Schickore & Steinle (2002) as saying that:

Design researchers choose to work in the 'context of discovery' rather than in the 'context of verification'.

These quoted descriptions of design research fit the way I worked with the observation and the data I collected in this part. Design research consists of a process of cycles concerning shaping a design, carrying it out in practice and analyzing how it works and why. It is an iterative process that binds theory and practice together.

The design can be materials, a form or a program – in my case an educational program for an in-service course - where the instructional design is of interest. Figure 3 illustrates how design research works (Gravenmeijer, 1994; from (Cobb et al., 2003), and how I used it in my study:

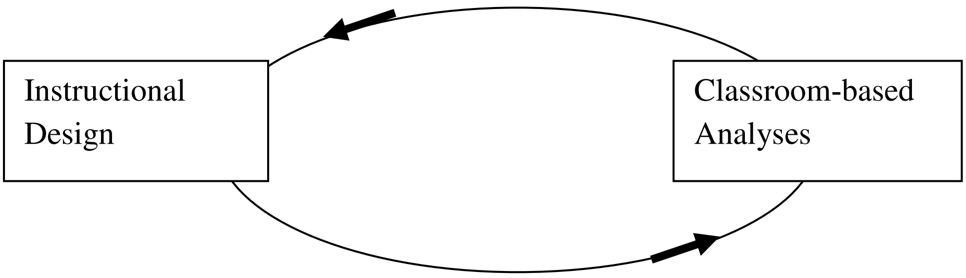


Figure 7: Design research is a continuous process between design and analysis. In this case, the classroom is an in-service classroom. The arrows symbolise the process from preparation of the design to a realisation. The processes of how the analyses of the classroom-based observations influence designing of new instruction.

The design-based research became a useful method when I analysed my data, and found that there was a need for a redesign of the in-service course, a design based on a new hypothesis developed from the analyses. I found the need for changing when I worked as a researcher, and through my research analysing of my data I decided what elements in the in-service course that should be developed and why. The developing was made by a combination of being the researcher and the teacher educator. I carried out the redesign as a teacher educator, collected data and investigated the new data as a researcher. The results were again treated from both positions.

3.3.2 'Theory' for design studies

One of the main ideas in design research is to develop theory, but the term 'theory' has many different connotations. Cobb et al. (Cobb et al., 2003) connect theory with design experiments in this way:

Design experiments are conducted to develop theories, not merely to empirically tune 'what works'. (p. 9)

What does it means that it should not only 'work' empirically but merely develop theories? What does theory mean here?
Niss (Niss, 2006) investigates how the concept 'theory' is used in mathematics education and what role it plays. He writes:

> *(…) it is not clear at all what 'theory' actually means in mathematics education. Nor is it clear where the entities referred to as theories invoked in mathematics education come from, how they are developed, what foundation they have, or what role they play in the field (p. 1).*

He gives further his own definition of the concept:

> *A theory is a system of concepts and claims with certain properties, namely:*
> *(…) an organised network of concepts (…) concepts are linked in a connected hierarchy (…) In the theory, the claims are (…) taken as fundamental (…) by means of formal or material derivation.*

About the roles and functions 'theory' plays, he says:

> *(…) different theories have different roles in research in mathematics education.(…) Some theories serve as an overarching framework from which the teaching and learning can be viewed and approached. (…) Some theories focus on organising a set of specific observation and interpretation of singular but related phenomena into coherent whole. (…) Some theories have the role of providing the terminology involved in a particular piece of research. (..) Some theories offer a research methodology, primarily for empirical studies. (pp. 8)*

In this dissertation, the word theory change explanation for different purpose to catch the different complexity that arises. The different uses concern:

- theory as different from practical empirical data collection or experience from practical teaching, where intuition plays a role,
- theory as a tools for interpreting data,
- theory as ideas, frameworks or results from research literature,
- theory as new hypotheses produced for the redesigns

The different theories became tools for me in different situations, and I will explain how the term 'theory' is used in each of the specific cases, where there is a risk of misunderstanding.

Cobb and diSessa (DiSessa and Cobb, 2004) explain their interpretation of theory in design research:

> *Theory can, however mean different things to different people. What kinds of theories can be produced by and can serve to further the aims of design experiments? Among the many possible criteria for types of theories, one stands out as critical for design studies. Theory must do real design work in generating, selecting and validating design alternatives at the level at which they are consequential for learning. (p. 80)*
> *(…) development of theory should be one of the primary goals of design research. (…) we contend that design experiments have generally been underdeveloped as contexts for the development of theory. (…) we concentrate on one distinguish process in developing the kind of theories in which we are interested, that of "ontological innovation" – the invention of new scientific categories, specifically, categories that do useful work in generating, selecting among, and assessing design alternatives.*

Even though we are in the field of a very complex human science, where theories cannot be tested in the same way as in classical science, we have a pressing need for theories, theoretical frameworks, concepts or ideas. The theories in design research must do the design work and new theories must be developed. That the theory must do some work has Niss in common with Cobb and DiSessa, a question for me is how I use the accessible theory to develop a new theoretical framework that works as a generator for the designs I try to improve, without only make it ‘works better’.

In a talk with A. diSessa about my results and how to develop hypotheses, he considered my work as a way into the ‘landscape of theory’.

My results are based on empirical data and analyses. I developed some educational hypotheses that were tested, reflected upon and revised through 3-4 cycles of new in-service courses. The conclusions from the results and the model developed are directed more to other teacher educators or researchers than to practising teachers. The theoretical framework developed in this study is based upon a kind of ontological innovation in the sense that it is able to do useful work in generating, selecting among, and assessing design alternatives in the area of in-service education.

3.3.3 Design research at LLD

As mentioned before, the STL group at LLD was interested in finding out how design research could benefit our different research projects and be flexible enough to allow for both design and development of theoretical insights. A consequence of that interest was that we discussed where our needs fitted into the design development and where we needed to rethink our definition of design research. We were asked to describe this process in a chapter for a book about design research (Kelly, 2007). The following two pages is a quotation from this chapter, where I describe my project in a model developed by the group (Ejersbo et al, 2007). The […] are my comments:

“In order to have a model that addresses the push and pull of the work flow in projects, we - the STL group at LLD - developed an `osmotic´ model as shown in figure 1 [of course it could look like a caricature, but it is useful in the model]. The model refers to the process of osmosis, because there is an inherent fluctuation between concentrating on designing and theoretical reflections. The osmotic model is not an instruction manual for doing proper research, merely a simplification of navigating between various aspects in the research process. The arrows are meant to show that there is flow, a dynamic osmotic force. The arrows are not indicators of a sequence or a chronology - rather they are phases of a research process, which seem to be necessary for maturity of a design research project. The model takes departure at the centre or ‘the problem’; and the optimal research process should then be understood as performing iterative and synchronous circle movements in both directions. Note that the word “artefact” should not necessarily be understood as material objects like an abacus or a game, it may just as well be learning strategies, organizational changes or other immaterial process descriptions, which serve as curricular end-in-view or inspiration for prototypes.

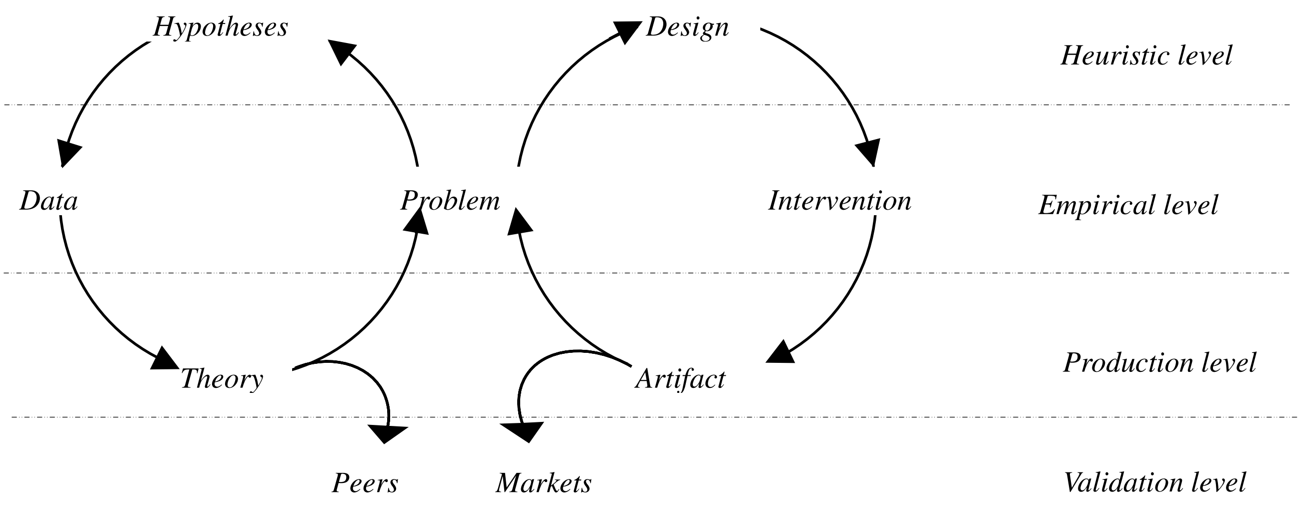


Figure 1:’Osmotic Mode’: Our current understanding of how to balance artifact and theory generation within a design research paradigm. The left circle mimics the traditional way of doing educational

research, where the main "customers" are the peers. The right circle mimics a normal production cycle, but with a much stronger involvement of user feedback. Ideally, a design research project moves in synchronous circular movements, starting from the centre and going in both directions. However this synchronicity rarely happens in practice.

In order to explain this very idealized and macroscopic model for conducting research, we break the model down into four steps or phases: a) from problem to design and from problem to hypothesis; b) from design to intervention and from hypotheses to data; c) from intervention to artefact and from data to theories; and d) from artefact to markets and from theories to peers.
a) From problem to design and from problem to hypothesis
Going from a problem at hand to a hypothesis/design entails making a move from the empirical level into the heuristic level - probably the most exciting but also most difficult part of doing research. A pre-requisite is that the researcher has a fairly good knowledge of existing theories about the theme. It also helps to have a sound scientific intuition when making a new hypothesis (a proto-theory) about how the particular problem could be confronted and possibly solved. In order to make this move, a researcher should be able to induce a solution, for example a change of practice. This requires a working knowledge of existing theories, existing artefacts, and design intuition.
b) From design to intervention and from hypothesis to data
It is on this level that design research has a great deal to contribute. Design research implies that the move from design to interventions is never linear; rather it is a circular, iterative process. There can be infinite loops of designing, intervention and redesigning. So, like Ptolemaios, we ought to draw small epi-circles into the figure, between "design" and "intervention" and between "hypothesis" and "data", in order to acknowledge this fact.
c) From intervention to artefacts and from data to theory
Single classroom interventions and follow up qualitative research are the prime activities for Design researchers at universities. But in order to maintain an ambition of infecting learning communities with new tools and new ideas, we need to create innovative instructional designs which are readily translatable and adaptable to many contexts. An aspect of this need is preparing the artefact for diverse contexts, and not to be satisfied with localized prototypes. It is an important ambition but presents some serious challenges and even obstacles.
d) From artefacts to markets and from theories to peers
In order to ensure successful interventions within educational practice, researchers should consider deployment just as important as theory and artefact development. However, there is cause for scepticism. The history of education reform shows us that very little of lasting effect has been produced by the educational design experiments to date (Williamson, 2003).
(…)
In addition to the dynamics in figure 1, four conceptual levels are identified. These levels are: the heuristic level; the empirical level, the production level and the validation level. The heuristic level relates to hypotheses and prototype design, where commonsense rules, intuitions and creative processes are mixed and used in order to increase the probability of finding a good candidate for further inquiry. The empirical level in contrast tries to systematize what can be known and what is unknown through well-established scientific operands of experiments and observations, verification, falsification and so on. The production level involves competencies such as organizing, framing, planning, synthesizing and sometimes delegating work.
Last but not least: the validation level is less in the hand of a researcher than of people or mechanisms that are used for authentication and dissemination.
Some final comments on the osmotic model – we, at the STL group, are proposing that one way to contribute to education reform in the future is to be extremely conscious about creating marketable products which are disseminated to the proper audiences. In this way, we can extend academic validation through peers by external evaluation and selection through users and markets. Thus, evaluation is threefold: peers, markets and user feedback. Beware that 'markets' should not be misunderstood as 'mass markets'. The word markets should be understood as the many different recipients, target groups and stakeholders of the artefacts in question. These stakeholders might be the relevant people who have never heard of you, but who might profit from your design efforts. Thus, in order to ensure successful interventions in educational practice, deployment should be seen as being equal in important to development theory and artefacts."

Within this structure, I can describe the process for the design and redesign of my study combined with my data in a new way: My first design of the in-service course was an artefact designed before the Ph.D.-course and before I knew all the problems that would arise. The different evaluations of the design, the classroom observations and the interpretation of the first data collected showed to me that there were some problems I had not been aware of. I needed to redesign the course. My hypotheses for this redesign were based on the data collected in the classroom, and concerned my knowledge of the teachers' knowledge and meta-knowledge and my implicit assumptions as the teacher-educator/researcher who designed and ran the courses. New data were collected when the redesign was implemented. Course 2005 can be viewed as the artefact evaluated by the market and the 'theory' is the results of the study that will be evaluated by my peers.

Figure 8 shows the process of my study:

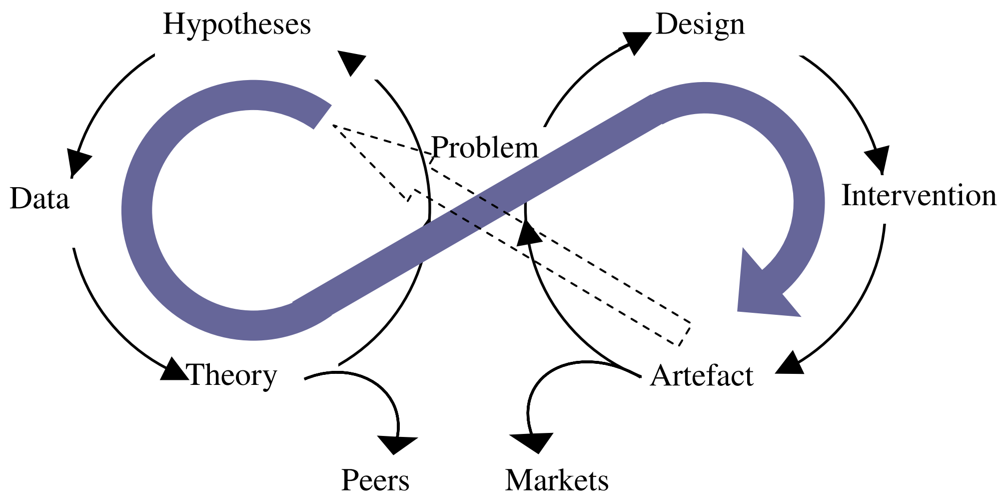


Figure 8: The methodological path taken in the project of teacher in-service training in using open problem solving in mathematics. The project started out with an ad hoc 'artefact' – the existing in-service course; this course has now been redesigned on the basis of the ethnographic data and theoretical refinements.

C-02 was the first in the study and the course was repeated three times during the study. This opportunity gave me the chance to do experimental design research and to generate and cultivate my hypotheses alongside the courses.

This framework helped and allowed me to consider the complexities of mathematics teaching, and of reflection and revision of the design and redesign of the actual model (Wood and Berry, 2003).

This intimate relationship between the development of theory and the improvement of instructional design for bringing about new forms of learning is a hallmark of the design experiment methodology. One of the distinctive characteristics is that the researcher deepens his or her understanding of the phenomenon under investigation while the experiment is in progress (Ibid). Here, the sequence of in-service courses constitutes the iterative process that ties theory and practice together, and the artefact is the design of those in-service courses and the underlying theoretical framework.

PART B: Pre-study

As has been stated, the methodological approach in this study is design research. The point of departures is Course 2002 (C-02) taken to be the initial artefact, which was used prior to the redesign of the in-service course. As a pre-study, I investigated C-02, and was examined to what extent and in what ways some of the participating teachers adopted teaching with open practical problems in their own classes. I looked for any problems the teachers might have and compared them with how those problems were addressed in C-02. I then developed a procedure for redesigning the course based on the data analysed.

I collected and interpreted data before, during and after C-02 as for the pre-study. The data concerned the participating teachers and C-02 itself. Teaching mathematics with open problems is both vital and difficult, and in this chapter I will specify some types of problems the teachers encountered, when they used open practical problems in their own mathematics teaching.

This part B comprises descriptions of some of the ways in which open practical problems are defined and dealt with in the literature on mathematics education research, characteristics of the four teachers as revealed in personal interviews and case studies consisting of the observations I made, when I visited their classrooms after the course. The last part of the chapter contains my interpretation and discussion together with a conclusion for the pre-study. I include the entire pre-study because the results obtained are important for understanding the redesign of the in-service course.

1. Open Practical Problems

The executive order from 1997 on the oral examination in mathematics after the ninth and tenth grades states that the task for the oral exam must be formulated as a practical problem, and recommends that it should be an 'open' problem. From the literature (Blum and Niss, 1991,Palm, 2002) we know that there is no universal agreement on terms like 'problems', 'model', 'authentic', 'application', 'practice', 'real life' 'open', etc. The term 'open practical problems' is not unambiguous either, but because requirements and recommendations from the Ministry of Education involve the words 'practical', 'problems' and 'open', I will look for the official ministerial explanation of, it if it exist, while also considering in order to how the different terms are lineated in the literature, finally clarify how I use the terms in this study, starting with the term 'problem', then the term 'practical' and last the term 'open'. Because the content of the in-service course is to master a requirement from the Ministry of Education, I will constantly take in the formulations from the official regulations and discuss how they could be understood.

The Danish project KOM (Niss and Jensen, 2002) lists eight competencies as main constituents in an answer to the question 'What does it mean to master mathematics?' Their definition of possessing a mathematical competence means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or could play a role. The eight competencies are (italics from the original, Niss (2003)):

1. *Thinking mathematically, mastering mathematical modes of thought, includes*
   - *understanding and dealing with the roots, scopes, and limitations of given concepts;*
   - *abstracting concepts, generalising results;*

- *distinguishing between different types of mathematical statements, e.g. definitions, theorems, conjectures, statements concerning single objects and particular cases;*
- *possessing awareness of the kinds of questions that are typical of maths, and insight into the kinds of answers to be expected;*
- *possessing an ability to pose such questions.*

2. *Problem handling, formulating and solving mathematical problems, includes*
   - *detecting, formulating, delimitating, and specifying mathematics problems, pure or applied, open or closed;*
   - *possessing an ability to solve problems, posed by oneself or by others, if desirable in different ways.*
3. *Modelling, being able to analyse and build mathematical models concerning other areas, includes*
   - *analysing the foundations and properties of existing models, and assessing their range and validity;*
   - *performing active modelling in given context, i.e. structuring and mathematising situations, handling the resulting model, drawing mathematical conclusion from it, monitoring and controlling the entire process.*
4. *Reasoning, being able to reason mathematically, includes*
   - *following and assessing others' mathematical reasoning;*
   - *understanding what a proof is (not) and how it differs from other kinds of reasoning;*
   - *understanding the logic behind a counter example;*
   - *uncovering the main ideas in a proof;*
   - *devising and carrying out informal and formal arguments, including transforming heuristic reasoning to valid proof.*
5. *Representation, being able to handle different representations of mathematical entities, includes*
   - *understanding (decoding, interpreting, distinguishing) and utilising different kinds of representations of mathematical entities;*
   - *understanding the relations between different representations of the same entity;*
   - *choosing, making use of, and switching between different representations.*
6. *Symbols and formalism, being able to handle symbolic language and formal mathematics systems, includes*
   - *decoding symbolic and formal language;*
   - *translating back and forth between symbolic language and natural language;*
   - *handling and utilising symbolic statements and expressions, including formulae*
   - *understanding the nature of formal mathematics systems.*
7. *Communication, being able to communicate, in, with, and about mathematics, includes*
   - *understanding, examining, and interpreting different kinds of written, oral or visual mathematical expressions or texts;*
   - *expressing oneself in different ways, and at different levels of precision, on mathematical matters to different sorts of audiences.*
8. *Tools and aids, being able to make use of and relate to the tools and aids of mathematics, includes*
   - *having knowledge of the existence and properties of different relevant tools and aids for mathematical activity (e.g. rulers, compasses, protractors, tables, centicubes, abaci, calculators, computers, the Internet);*
   - *having insight into the possibilities and limitations of such tools;*
   - *reflectively using tools and aids.*

This way of characterising what it means to master mathematics differs quit a bit from the way mathematics is characterised as a set of skills through examination requirements and expressed in terms as factual knowledge and technical skills. I use the word 'competence' in accordance with Niss and Jensen (2002), where work with open practical problems is embedded partly the problem-handling competence and partly in the modelling competence, when the problems are applied and typically open. I will therefore rely on these competencies in my definition and description of open practical problems and use of the modelling competence when establishing the basis for a solution for the problems. Furthermore, as it is difficult to isolate the competencies, working with open practical problem solving includes activating other competencies as well.

Mathematical communication is another term that is often not adequately explained. 'Oral mathematics' is not understood in the same way by every teacher and the teachers' communication when the pupils' working on open problems was not as reflective as I expected before my study. Therefore mathematical communication will be examined as well along with a clarification of what I mean by this term.

1.1 Practical Problems

The recommendation concerning the oral exam (1998) defines a practical problem as follows:

> *A practical problem could be a situation from daily life such as opinion polls, a skiing holiday etc. It might also be something specific and hands-on like a cube, or something the pupils create themselves during the examination, for instance a model. (p. 26, my translation)*

It seems that a 'practical problem' in this interpretation could be several different things, such as a problem from daily life, a hands-on object, or a model – what kind of model is not explained. A practical problem can be nearly any kind of task. What kind of context to use for which kind of pupils is another question. Each year, the Ministry of Education publishes an evaluation of the written and oral exams. In the 2002 edition, it is stated that the best problems are those that contain some 'local hints', because these hints provide a background for pupils and help them apply a methodology they are familiar with. This result seems to confirm that successfully real life problems depend on the understanding the students have of the context involved in the tasks. I will leave out the 'hands-on problems like a cube', because we mostly worked with situations from daily life on the in-service course. It caused enough troubles to find problems that were a) open and b) had a kind of authentic application for the solvers, and c) qualified by the use of mathematics to the solution of the problem.

The first thing is to determine what a 'problem' could be. Blum & Niss (Blum and Niss, 1991) p. 37) define a problem as

> *…a situation which carries with it certain open questions that challenge somebody intellectually, who is not in immediate possession of direct methods/procedures/algorithms etc. sufficient to answer the question.*

A problem is only a problem if it is a problem to somebody. In this case it should be a problem for a pupil in ninth grade, but it should only be a problem in a mathematical sense, it should not be a problem to read and understand the text. The problem in this context should be written as a task in a way that the pupil understands, what the problem to solve is, and that it is a problem that could be solved with skills that the pupil is expected to manage. It means that the content of problem and the way it is formulated are two different things. It is said that the problem should be practical as well, which I understand as to be applied in some sense. As to an applied mathematical problem, Niss and Blum (ibid) characterise by stating that

> *…the situation and the questions defining it belong to some segment of the real world and allow some mathematical concepts, methods and results to become involved.*

Practical has as well many understandings. In the research done by Palm (Palm, 2002), he points out that the notion of authentic problems is described very differently, often within the same publication. What real world are we talking about when we think of the term 'practical'? Should it be authentic or should it only look like? What happens with the pupils' motivation if the declaration of the task doesn't fit into how they understand the problem?

Palm and Burman (Palm and Burman, 2004) analysed how real life problems were used in tasks in mathematics assessment at upper secondary level in Sweden and Finland. The authors draw attention to the fact that many mathematics tasks referring to real life were so manipulated when they appear in textbooks or in the written exam paper that they have lost all relevance. Palm and Burman showed that such pseudo-realistic tasks could even have a negative impact on learning, because the pupils felt cheated.

In their investigation, Palm and Burman classified 'realistic tasks' according to eleven parameters through simple yes or no questions (except for no. 6, which has three possible answers); simulated situation means the task situation as described in the text or the written exam, whereas the 'real life situation' belongs to is the reality outside the classroom that has been used for the simulation:

1. Could the event described in the school task happen in real life beyond school? (Event)
2. Have the questions or assignment been - or might they be - posed in a real life out-of-school task situation? (Question)
3. Is there a purpose in solving the school task resembling the purpose in the corresponding real life situation? (Purpose in the figurative context)
4. Is the same kind of information available in the school situation as it would be in the simulated real life situation? (Existence of information/data)
5. Is the information provided in the school task 'realistic' in the sense of being identical to or very close to the corresponding data in the real life situation? (Realism of information/data)
6. Are the subjects, objects, and places in the figurative context specific or general? (Specificity of information/data)
7. Is the terminology, sentence structure, amount of text etc. used in the presentation of the school task similar to the corresponding simulated real life situations? (Language use in the task presentation)
8. Are the available external tools such as calculators, computers, maps etc. the same in the school situation as in the simulated real life situation, and are they related to the relevant competencies for solving the task in the two situations? (Availability of external tools)
9. Does the task solver have the same kind of helpful hints, e.g. solution methods and types of answers required, in the school situation as he or she would in the real life situation? (Guidance)
10. Do the available solution strategies of the majority of most students match those of the persons solving the tasks in the simulated situation? (Availability of solution strategies)
11. Are the solution requirements in the school task the same as in the simulated situation? (Solution requirements)

With this analytical tool it is possible to investigate tasks claimed to be 'real life problems' and see where they fail or succeed with respect to there criteria.

In the present text, I use Blum & Niss' characterisation of a problem that a problem should challenge to be understood as a problem and 'authentic problems' refer to problems that could exist in a practice or subject area outside mathematics, and with solutions that should be authentic to people there.

The annual evaluation from the ministry points to 'authentic' as local hints and to 'practical' as 'both hands' on tools or a situation from 'daily life'. One example of an open practical problem copied from the Ministry of Education (1998, p. 28, my translation) is the following 'Weather Problem':

How was the weather?
Weather is always on our mind – holiday weather particularly so.
We tend to believe that the weather during last year's holidays was quite out of the ordinary.
The elderly often claim that the weather was radically different when they were young; summers were endless, warm and dry, while winters were freezing and you had to dig your way out of your house – and Christmas was white every year!
Choose a couple of periods for comparing. Base your comparison on the following information from a weather database: You are free to decide how you will correlate the numbers.

Data om vejret

Nedbør (mm)	1991	1992	1993	1994	1995	1996
jan	41	43	70	64	78	1
feb	21	22	19	52	41	18
mar	23	65	13	75	49	6
apr	42	62	13	23	33	6
maj	19	21	27	26	46	64
jun	72	1	15	56	40	32
jul	44	37	65	4	12	21
aug	26	99	44	104	13	43
sep	54	21	95	139	62	36
okt	29	57	66	45	51	50
nov	49	74	28	43	42	67
dec	46	23	82	49	9	21

Temperatur (°C)	1991	1992	1993	1994	1995	1996
jan	3,6	4,1	3,3	3,7	1,5	−0,7
feb	0,3	4,9	2,5	0,3	3,9	−1,7
mar	5,8	5,8	4,9	4,9	4,8	1,7
apr	8,7	8,1	10	9,9	8,6	10,4
maj	12,5	15,7	15,9	13,3	13,1	11,3
jun	13,9	21	17	16,3	16,6	16
jul	20,2	21,2	17,5	23,2	21,3	18,2
aug	20,4	18,6	17,2	20	23,2	22,1
sep	16,8	15,7	13,2	14,8	15,8	14,8
okt	11,3	8,9	9,2	10,2	13,4	11,5
nov	6,8	6,7	3,6	8	5,2	6,4
dec	4,5	4,3	3,1	5,3	−0,1	1,3

Nedbør (mm)	1891	1892	1893	1894	1895	1896
jan	36	51	32	37	26	15
feb	16	20	38	42	15	3
mar	58	9	20	41	44	42
apr	26	25	12	22	13	31
maj	51	57	21	45	27	32
jun	19	110	15	39	41	21
jul	66	13	81	67	87	42
aug	145	63	75	84	77	111
sep	36	49	57	41	10	67
okt	42	62	76	58	82	82
nov	27	36	35	33	74	13
dec	27	36	35	33	74	13

Temperatur (°C)	1891	1892	1893	1894	1895	1896
jan	−2,1	0,5	−4,1	1,3	−0,3	1,8
feb	2,1	1,1	−0,5	3,6	−3,7	3,2
mar	2,2	3,1	5,6	7,3	3,1	5,2
apr	6,6	9,1	10,3	10,5	10,1	8,6
maj	12,9	13	13,5	13,3	15,3	14,4
jun	16,9	15,7	17,9	17,3	17,9	20,6
jul	19,2	18,2	20	20,5	17,8	20,4
aug	16,6	18,4	19,4	17	18,5	17,6
sep	16	14,8	14,3	13,6	16,7	15,1
okt	12,1	9,8	11,3	8,8	9,7	10,5
nov	4,9	5,3	5	7,7	6,4	4,6
dec	3,9	1,1	4	4,5	1,8	1,4

The figures are from measurements performed every day at 6 p.m. Precipitation is the amount of rain that has fallen from 6 a.m. to 6 p.m.

Sommervejret i ferien (juli måned)

	1995			1996		
Dag	Nedbør (mm)	Temperatur (°C)	Vindhastighed (meter/sek)	Nedbør (mm)	Temperatur (°C)	Vindhastighed (meter/sek)
1	0	18	2	0	14	3
2	0	18	3	1	13	3
3	4	13	3	6	15	5
4	0	15	5	6	17	4
5	1	15	7	0	17	6
6	0	21	8	0	15	4
7	0	22	5	0	16	4
8	0	21	6	0	16	6
9	3	20	4	0	18	5
10	0	19	4	0	19	4
11	0	20	6	0	18	2
12	0	22	7	0	19	2
13	0	24	5	0	17	3
14	0	20	3	0	21	2
15	0	24	3	0	16	1
16	2	20	2	0	20	0
17	0	21	0	0	17	1
18	0	22	2	0	18	5
19	0	23	5	0	18	5
20	0	25	6	0	21	4
21	0	26	8	0	20	1
22	0	21	10	0	24	4
23	0	20	8	0	25	2
24	2	20	7	0	20	5
25	0	21	4	0	24	3
26	0	22	4	0	21	5
27	0	23	2	7	16	7
28	0	25	0	0	18	9
29	0	26	1	0	18	6
30	0	27	2	1	16	8
31	0	27	2	0	17	7

Tallene i tabellen angiver målinger, der er foretaget hver dag kl. 18.

Nedbøren er den mængde regn, der er faldet fra kl. 6 til kl. 18

Vindhastighed meter/sek	
0- 1,5	Stille
1,6- 5,4	Svag til let vind
5,5- 7,9	Jævn vind
8,0-13,8	Hårdvind
13,9-24,4	Kuling
24,5-	Storm eller orkan

Ministerial recommendations for the work with this task include: elaborating a conversation about the concept of 'average', tendencies and forecasts, and about the choice and use of graphical representation compared with numbers and oral descriptions. It is not obvious what kind of data we see in the 'precipitation' and 'temperatures' from 1891 to 1896 and from 1991 to 1996;

whether it is an average for a month or a day. It is an open question, whether this task is in accordance with the required to be open or authentic as a practical task.

In the Act about mathematics, we find the justification of 'practical' problems:

> *…the students become able to understand and use mathematics in contexts relating to everyday life, social life and natural conditions".* From the syllabus recommendation, p. 83*: "Practical use has always been the reason for school mathematics*" p.84*: "There is no doubt that the work with practical problems in the mathematics teaching motivate the students…*" p.65*: "The purpose with mathematics in school is not to make the pupils into mathematicians. The aim is to contribute to the personal development of the pupils as well as to give them experiences of how mathematics can empower them to solve problems in practice." (My translation)*

To sum up, the purposes of open practical problems are:

- They motivate pupils to work more with mathematics in school, and
- If school mathematics includes a practical context in the tasks, the pupils will know when and how to use mathematics in context outside the school when needed.

These purposes are hypothetical and not supported in the research literature. The expression 'to be practical' has different connotations. In this study, I have chosen to interpret 'practical problems related to daily life' as being equal to 'real life' problems, again using Blum & Niss' characterisation. The tasks used on the in-service courses were mainly 'real life' problems with a touch of authenticity, which means that it was intended to be as authentic as possible and still be a mathematics task. When I refer to practical problems in what follows it means 'real life problems', i.e. authentic problems from outside mathematics.

The complexity of 'real life' means that there are countless mathematical problems to solve, but the mathematical processes are often hidden, which is reflected in the so-called 'relevance paradox':

> *Even though mathematical knowledge is highly relevant in and to society, many, if not most, people have increasing difficulties at seeing that mathematics is relevant to them, as individuals. (Niss, 2003)*

This challenge requires that the mathematics teachers are or become able to see how mathematics is relevant to them and further to their students. Mathematics teachers use textbooks as a primary tool for their teaching. In many of these textbooks, and in the written examination papers in mathematics, it is customary to use different kinds of pseudo-realistic tasks; therefore it is difficult for the teachers to provide relevant models for producing authentic open practical problems from real life. In this dissertation, pseudo-realistic tasks are understood as construed from unauthentic references. An example of a pseudo-realistic task taken from Dowling (1998, here Skovsmose, 1999, p. 7) is this:

> *Shopkeeper A sells dates for 85p per kilogram. B sells them at 1.2 kg for £1. (a) Which shop is cheaper? (b) What is the difference between the prices charged by the two shopkeepers for 15 kg of dates?*

It is not a realistic task, rather is it a mathematical task dressed up with shopkeepers and some dates, where the shopkeeper is an 'A', a mathematical sign rather than a person. The category of pseudo-realistic tasks contain many different kind of task some more realistic than pseudo and vice versa, but mathematics textbooks are full of them.

'Realistic mathematics', as described in the Realistic Mathematics Education (RME) project, of the Freudenthal Institute in the Netherlands, does not necessary refer to real life problems. The following quotation defines the distinction between realistic and 'daily life real':

> *'Realistic' means 'experientially real', and not always 'daily life real' (Gravemeijer & Drijwers, 2004).*

These experientially real tasks called 'realistic' can be pseudo-realistic tasks in my categorisation. Mathematising is one of the main aims in developing RME and is defined and explained as the process

> *…that involves solving problems, looking for problems and organizing a subject matter resulting from prior mathematizations or from reality. (Gravemeijer et al., 2000)*

Van Oers (van Oers, 2000)defines it like this:

> *According to Freudenthal, mathematics is basically an activity of mathematizing: that is, organizing a (concrete empirical or abstract mental) domain, representing it with help of symbols , finding problems, solving problems, and experimenting with symbolic means (as in a thought experiment), in order to become better acquainted with the properties of the domain under consideration. (2000, p.135)*

Mathematising is understood as a 'horizontal and vertical mathematisation' and can be represented by the following model, taken from a presentation by Gravemeijer and Drijers at Learning Lab Denmark (DPU) 2004:

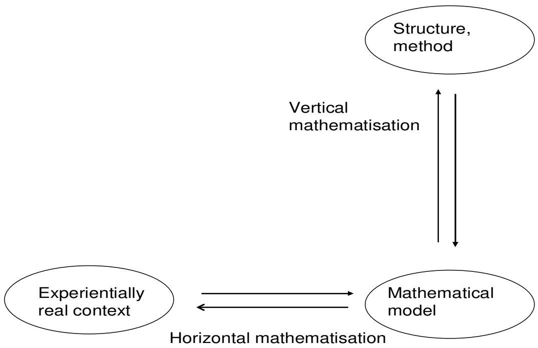


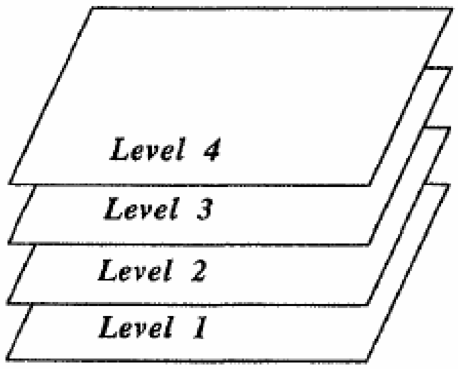
Figure 9: Horizontal and vertical mathematisation

Horizontal mathematisation takes place when pupils work with informal mathematics in a familiar context, 'taken-as-shared' ways of speaking, and symbolizing their starting point's problems. Vertical mathematisation is explained as when informal mathematics becomes the subject of the work of symbolizing the problems and of discussion for further mathematising that can lead to generalisations (Treffers, 1987; here Gravemeijer, 2000). 'Taken-as-shared' is when participants in a classroom discussion are convinced that a meaning is shared and the participants know that this is an assumption for the moment (Voigt, 1996) p. 33). It can be confusing that both processes are called mathematisation.

Mathematising in this interpretation is a key process in mathematics education for two reasons: It is the major activity of mathematicians, and it familiarises students with a mathematical approach to everyday settings (Freudenthal, 1991). RME is rooted in Freudenthal's interpretation of mathematics, where the classroom micro culture is investigated through observations of the individual pupil. The transposition of ideas into the classroom community depends on the conjectured learning process.

The following model describes the idea as understood in RME:

The term model (…) can refer to a task setting or to verbal description as well as to ways of symbolizing and notation. (…)In RME, the term model is understood in a dynamic, holistic sense. As a consequence, the symbolizations that are embedded in the process of modeling and that constitute the model can change over time. (Gravemeijer et al., 2000, p. 240)



Level 1: Activity in the task setting, in which interpretations and solutions depend on understanding of how to act in the setting (often out of school settings)
Level 2: Referential activity, in which models-of refer to activity in the setting described in instructional activities (posed mostly in school)
Level 3: General activity, in which models-for make possible a focus on interpretations and solutions independently of situation-specific imagery
Level 4: Reasoning with conventional symbolizations, which is no longer dependent on the support of models-for mathematical activity

Figure 10: A copy of the 'Levels of activity' (ibid: 243)

One of the key points in the model is the process of a developmental progression divided into four levels of activity. In this model, Gravemeijer et al. distinguish between 'model-of' and 'model-for'. A model-of at level 2 is explained as the description of first informal model situation (horizontal mathematisation), while a model-for at level 3 is the mathematising of the results from the informal modelling activity using more mathematically structured reasoning (vertical mathematisation). One way to illustrate the concepts model-of/model-for, which is used as example in the article, is a situation in which the pupils set out to determine the number of busses needed to transport a certain number of passengers. To start with, the pupils may solve the problems through repeated subtraction; this is a model-of the situation because it is the first informal model of the situation. Later, the class may discuss ways of using multiples for solving the problem; here the discussion is about the mathematics taken-as-shared, and the arguments used are more structured terms of method. This shift in the discussion from explicitly talking about the situation to discussion of and reasoning about the mathematical relations involved is called a model-for.

Another conceptualisation of the mathematical modelling process, which is closely related to mathematical applications (Blomhøj and Jensen, 2002) (Christiansen et al., 1997) is presented by Blomhøj and Jensen (2002, p. 5) as follows:

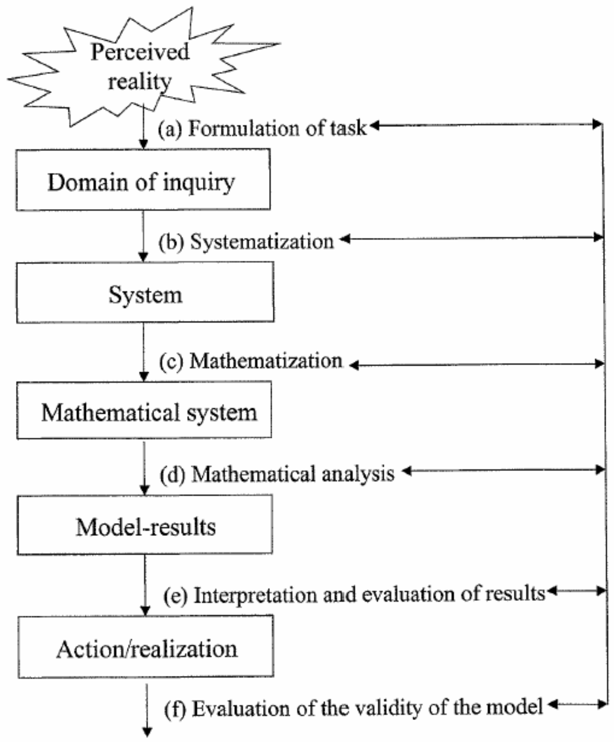


Figure 11: A copy of the graphic representation of a mathematical modelling process (ibid)

This model shows an overview of the total process, but not the process of transition from one activity to the next. This modelling process starts in practice and ends in practice after a horizontal mathematisation, in contradiction with the RME model which ends in mathematical symbolism (vertical mathematisation). The two models have different foci. The RME process does not focus the results of the problem, rather the mathematical process, while Blomhøj's and Jensen's model focus on how the results can be used in the perceived reality. In the latter model the validity of the model is a part of the process, which it is not in the RME models.

Inspired of these different models, a colleague and I - in 1997 - collated the experiences from the first in-service courses (1996) in the form of a booklet (Ejersbo and Andersen, 1997). We collected the best tasks developed by the participating teachers and formalised a kind of procedure to be used as a strategy by the pupils during the exam. The model of this procedure is inspired by both the 'levels of activity' and 'mathematical modelling'. Our model has five steps, where the first three (0-2) depend on the task context, while the remaining two steps (3-4) give the pupils the possibility to generalise and reshape the relationship between practice and theory; these steps are independent of the task content.

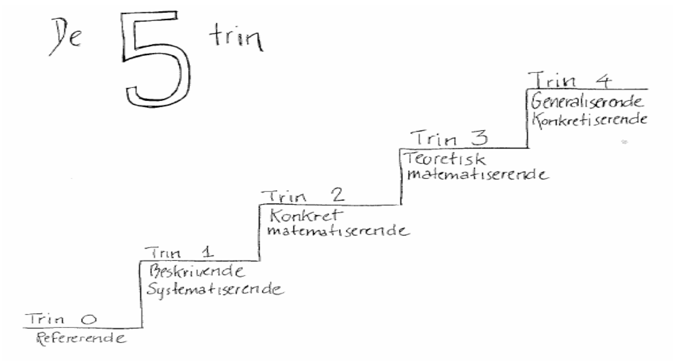


Figure 12: The five steps copied from the booklet 'Ideas for the oral examination in mathematics, Open tasks' (ibid: 7).

The five steps are as follows:
Step 0: Referring: The pupils only relate to the context with mathematically unreflective recognition.
Step 1: Describing and systematising: The pupils make preliminary quantitative and qualitative descriptions of the material in the task context.
Step 2: Concrete mathematising: After a discussion, the pupils consider their choices to questions and apply (hopefully) the relevant mathematics related to the material in the task context.
Step 3: Theoretical mathematising: The pupils reflect through mathematical communication on the mathematical concepts used in solving the task.
Step 4: Generalisation and concretising: The pupils consider examples of the mathematical theory in other contexts.

These steps are meant as a strategy to help the pupils organise their time at the oral examination, and as a help to the teacher who has to assess the performance. Furthermore, they support teachers in developing mathematics communication among and with pupils, and in informing them about the criteria for assessment. Step 2 in the 'five steps' has many features with level 2 in the 'levels of activity' (horizontal mathematisation), while step 3 shares similarities with level 3 in the 'levels' (vertical mathematisation). The given task is based on a realistic problem and the steps include a mathematical modelling as well. After the description (step 1) the concrete mathematising calls for a model to use. The procedure is developed to an exam situation where the teacher asks questions to test the pupil's skills. These steps turned out to help the teacher asking relevant questions about both mathematics and the mathematical modelling process, and the pupils were encourages to work with modelling as well as model-for. In that way the steps turned out to be a help in the sense that it constituted limits for the process, which was often too open; it was the problem that should be open, but often it was confused with the process as well. Using this model, the teacher could now ask more focused questions in a limited area and the pupils had a scaffold to lean on. Both the 'levels' and the 'five steps' describe a kind of epistemological development in working with mathematics based on real life, while the modelling describes the process that goes on when it happens. At the in-service courses, the 'teachers' are still interested in this procedure, which tells me that it works as a usable tool.

If we picture a group of pupils solving the 'weather task', we could examine the following question, where I give my answer or comment after each question:

- What kind of problem do we have here?

We have a description of a situation, but not a real problem. There is no question rather an instruction for what to do. The context belongs to some segment of the real world and allows for some mathematical concepts, methods and results to become involved.

- Which competencies could be used?

It is possible to use all eight competencies. The task can be discussed on several levels, and the actual competencies implicated depend on the further questions asked by the pupils when they solve it.

- What kind of practical problems does the task involve?

To investigate the weather is using a practical situation in the real world, but the tables are not 'realistic' and there is no problem to solve.

- How authentic is the problem?

Following the eleven parameters, the task already fails in the first or second parameter, because the situation described is pseudo-realistic and there is no question.

- What levels (RME) could we imagine in the work?

It is possible to go through all four levels, but again this depends on the further questions the pupils are able to ask or pose.

- What kind of mathematical modelling is possible?

The modelling process requires a problem that needs mathematics to be solved or investigated. Again it depends on the questions the pupils ask whether it makes sense to engage in a modelling process.

- Is it possible to use the five steps?

It is possible to follow the steps, but again this depends on the questions the pupils will ask and how well they will answer their own questions.

In this very open task, we see that the pupils own questions play a crucial role for how the solution of the task will be assessed. This kind of tasks requires that the pupils are able to formulate mathematical questions or problems that are relevant in the given context.

1.1.2 Summary of 'practical problems'

In the present text, a 'practical problem' related to mathematics is defined as a problem that challenges somebody intellectually in the real world, and which allows some mathematical concepts, methods and results to become involved; an authentic problem is characterised by potentially having solutions that would appear authentic to people in that situation. Furthermore, it is noted that pseudo-realistic problems are often called 'practical problems' in school settings.

The different tools or analytic frameworks presented are:

- The eight competencies, the Danish KOM project (Niss & Jensen).
- Criteria for a task to be authentic (Palm & Burman)
- A model for 'Levels of activity' (Gravemeijer et.al.)
- Mathematical modelling processing (Blomhøj & Jensen)
- The five steps: A strategy for solving a practical task (Ejersbo & Andersen)

For the teachers who need to learn how to formulate and teach through practical problems in mathematics, the conclusion is that there is not much help to be found in the ministerial recommendations to fulfil their own requirements. This means that the teachers need to be aware of the different approaches and to be able to choose the 'best one' from an unclear mess of parameters.

1.2 Openness

The ministerial recommendation (Undervisningsministeriet, 1998) dictates that the practical problems should furthermore be of an ‘open nature’. This feature is supposed to facilitate demonstration of different aspects of the pupils’ mathematical knowledge. Moreover, four different ways for a task to be open are defined in the recommendation: The description in the beginning, different solving methods, different strategies, and several solutions in the final outcome. There is no further explanation or references for this recommendation. ‘Different methods’ and ‘different strategies’ are difficult to tell apart. When I asked people from the ministry what the difference was, I never got a clear answer.

In a research report from the PME (Psychology of Mathematics Education) conferences 1993-96 edited by Pehkonen (Pehkonen, 1997), he gives an overview of how the use of open problems is explained in different mathematical cultures. Pehkonen includes the so-called ‘open approach method’ developed in Japan, the use of investigation developed in England and from the Netherland the ‘realistic mathematics’, is approached as open problem-solving. As for single term that includes the different meaning and expressions, Pehkonen proposes we call all the different types of problems ‘open problems’. He categorizes the different types of open problems in a matrix that contains closed problems as well. He explains the difference between a closed and an open problem as follows:

A problem is closed if the starting conditions and end state are precisely defined. If the starting situation and/or the goal situation are open, i.e. are not closed, then we have an ‘open problem’. (p. 5)

Goal situation / Starting situation	CLOSED (i.e. exactly explained)	OPEN
CLOSED (i.e. exactly explained)	Closed problems	Open-ended problems Real-life situations Investigations Problem fields Problem variations
OPEN	Real-life situations Problem variations	Real life situations Problem variations Projects Problem posing

Figure 13: The classification of problems according to their starting and goal situation. (ibid: 9)

The starting situation, I will say, is how the problem is formulated in the text available and the goal situation is specified by the expectation the answers should meet. The strategies to reach the goal are not represented, but it is obvious that these strategies can be in multiple.

As I see it, ‘Real-life situations’ and ‘problem variations’ are placed in three different ‘boxes’, which shows that the matrix classification is not categorical. The expression ‘real-life situations’ is explained in the section of ‘practical problems’. ‘Problems variations’ covers different problems without specific questions or problems treated as the ‘what-if’ method, which will be explained later together with the problem-posing methods.

Using open-ended problems in the classroom was, according to Pehkonen, developed in Japan in the 1970's. Japan has a tradition for working with open problems, which are called open-ended problems. Becker & Shimada (Becker and Shimada, 1997), p.1) denote open problems as:

We propose to call problems that are formulated to have multiple correct answers 'incomplete' or 'open-ended' problems.

Becker and Selter (Becker and Selter, 1996), p. 529) describe the Japanese understanding of open problems this way:

The open-ended teaching uses problems that do not have only one answer or one approach to finding the answer that is either correct or incorrect. Instead, the teaching proceeds by using a multiplicity of correct answers or approaches to provide experiences in findings something new in the process, through combining children's own knowledge, skills and mathematical ways of thinking. (…) Since Japan has a tradition of using problems with a unique answer, but many ways to find it, altogether the Japanese work represents an interesting and important use of 'openness' of problems in mathematics teaching, as
- the process is open
- the end-products are open, and
- the ways of formulating problems are open.

Here we have only three different ways instead of the four used in the Danish ministerial recommendation, but the description of the Japanese way is not only about the open tasks, but also about open-ended teaching or the 'open approach'. The discussion about how different strategies can be used to solve a problem and how new questions can emphasize the process is a main idea in Japanese mathematics education. That the 'process is open' means that there are many paths to the same results. They (ibid p. 526) give the following example to be used after the pupils have begun to learn addition of whole numbers:

How can you find the answer to 8 + 7?

As a follow up to this kind of problem, further questions could be asked to investigate kinds of problems the pupils could solve from this knowledge.

The use of 'investigations' became popular in England in the 1970's as well. It is a kind of open problems, which concern mathematics-ladder situations (William, 1994,Morgan, 1997). Similar to 'real-life situations', this could be to arrange transportation in busses for a group of children, or it could be a sequence of problems that are connected to each other in a so-called 'problem field'. The investigation draws upon the teachers' experiential knowledge of how children learn and how teacher interventions can help them learn. Furthermore, investigations depend on the teachers' ability to ask powerful open questions, which means questions that encourage the pupils to pose and discuss further questions. In their work with investigations and solving open problems, Watson and Mason (Watson and Mason, 1998) find it necessary to go beyond the simple categorization of just open/closed tasks and ask questions that promote thoughts about the structure of a concept and claim:

Thus most 'closed' questions can be opened up, and many apparently 'open' questions are nevertheless constrained. (…) Openness and closure of questions are relative to the teaching context.

Working with open problems is, in these cases, to be understood as a process which goes on in a mathematical communication process or context.

As an example of the term 'Problem-posing', Brown and Walter (Brown and Walther, 1993,Brown and Walther, 1990) describes problem posing as a procedure using a 'What-if-not' strategy to go on asking questions. The idea is to let the pupils pose problems on their own. In their concept ‘Problem Posing’, it is explained that the term covers a special way to ask questions about standard topics to give the pupils a deeper understanding: It starts with the choice of a task which all the pupils are able to define and ‘attack’; the next step is the real investigation, which starts with listing attributes, asking ‘what-if-not’ questions and analysing all the problems that arise from that. The pupils follow this procedure, and again the process depends on the way the pupils are taught to ask questions as well as how the teacher asks questions. Goldenberg (Goldenberg, 1993) expresses the difficulties in finding good problems like this:

I most strongly sensed the insufficiency of finding good problems for others after I had embarked on an exploration because I was curious. Even though I had had quite an exciting time of it, I felt seriously let down by the realization that finding such ‘good problems’ for students to do was somehow missing an essential ingredient. (p. 31)

The working process depends on the collaboration between the teacher and the pupils, which pertains to both problem-posing and the Japanese method. There are of course many other ways to pose problems, which deals with other questions than just ‘what if not’.

The concept ‘project work’ that Pehkonen refers to is a Finnish version, which is not explained further. In Denmark, ‘projects work’ has been used for several decades at many educational institutions. Skovsmose (Skovsmose, 2000) uses project work in explaining another way to differentiate between open and closed problems when he introduces ‘landscapes of investigation’. He uses a landscape metaphor which corresponds to the ‘landscape’ that project work is situated in. Skovsmose explains it this way:

Project work represents a learning milieu, different from an exercise paradigm.

Figure 14 is Skovsmose’s matrix of ‘learning milieu’ combined with his explanatory text:

	Paradigm of exercises	Landscape of investigation
References to pure mathematics	Exercises in ‘pure mathematics’	A landscape of investigation comprised by numbers and geometry
References to a semi-reality	Exercises with references to a semi-reality	Invitation for the pupils to explore and explain a semi-reality
Real-life references	Real-life based exercises	Project work

Figure 14: Skovsmose’s matrix: ‘Milieus of learning’ - combined with explanation

The reference to a semi-reality is explained in the example above (Dowling, 1998, here Skovsmose, 2000, p. 7). Tasks that fit into the category of ‘Landscape of investigation’ and ‘real-life references’ are called ‘Project work’. This kind of project work is closely related to authentic problems, where mathematics plays a role in a critical education (Skovsmose, 1994).

One of the pitfalls in working with projects in mathematics is the ‘learning paradox’ (Wistedt, 2001) p. 224):

The pupils have to know at least something about the project field and mathematics in order to ask relevant questions for how mathematics can be used.

The Danish Act calls for ‘Project work’, which means that every pupil in ninth grade must participate in project work, and one week is set aside for this project. In project work, pupils formulate their own contextual problems. These formulations of the problem guide them through the work to find answers to the problem. Mathematical modelling, as explained in figure 11 (Blomhøj & Jensen, 2002), could be used in such process, but the use of mathematics seldom finds its way into project work (Ejersbo, 2001). Problems or tasks used for project work in lower secondary school often have ‘an open description in the beginning presentation’ or some freedom in ‘the way of formulating the problems’. In this kind of openness, the pupils need to formulate problems that could also be closed problems. Many Danish teachers are familiar with the tradition of project work, but they are not experienced in using mathematics as qualifying factor in this work.

One often mentioned benefit of working with open problem solving is that the pupils are allowed to come up with solutions that the teachers did not expect, but for this to achieve the objective it is important that the setting is friendly, so that the pupils dare to suggest solutions, and it is necessary that the pupils learn how to investigate an open problem.

As in Denmark, open problems are used for assessment in ‘The Californian Assessment Program’ (1989, p. 1; here (Sullivan and Clarke, 1991), p. 43). In this program, they identify specific benefits associated with the use of open-ended questions in assessment:

- *Open-ended questions provide students [with] an opportunity to think for themselves and to express their mathematical ideas [in ways] that are consistent with their mathematical development.*
- *Open-ended questions call for students to construct their own responses instead of choosing a single answer.*
- *Open-ended questions allow students to demonstrate the depth of their understanding of a problem, almost an impossibility with multi-choice items.*

It is again pointed out that open-ended problems should be combined with good questions.

Sullivan and Clarke (1991, p. 16) define good questions as possessing three features (italics by the authors):

1. *They [good questions] require more than recall of a fact or reproduction of a skill.*
2. *Pupils can learn by doing the task, and the teacher learns about the pupils from the attempt.*
3. *There may be several acceptable answers.*

Again, this means that the teacher must be skilled enough to ask good questions that can encourage the pupils to investigate settings and look for patterns, and that the teachers themselves have a thorough knowledge of mathematics. And again the teachers’ abilities to ask powerful questions are emphasized as key factor for teaching mathematics in this way. The issue of how to do this seems to be one of the primary hurdles for using open problem-solving in the teaching of mathematics.

The Danish requirements to the oral exam reflect an international consensus regarding the nature of mathematics teaching and learning, in which both the tasks and the organisation play crucial roles. Therefore, it is common at the kind of in-service course studied in this dissertation to discuss and negotiate both the tasks and how the teaching is to be organized in the day-to-day teaching and how the examination is conducted. The models referred to have so far not been a categorical division into open or closed problems and all the examples show that the questions following the initiating task are important for the process. The following matrix is one way to categorise problems into open and closed problems:

Expected answers / Starting task-text	CLOSED (given)	OPEN (many possibilities)
CLOSED (i.e. precisely defined)	The text makes it clear that definite answer is required, which could contain only one, none or several solutions	If the condition of the task context changes from being defined say in R to for instance, being defined only in N, the solution could involve many possibilities
OPEN (not clearly specified in advance what acceptable answers could be)	No tasks	Tasks, inviting investigation, further questions or require the need to make decisions

Figure 15: A categorical matrix to divide tasks in open/close, based only on the initial text

Compared with Pehkonen's classification of open problems, my classification is categorical and concerns the formulation of the task-text, which is what the pupils have as their starting object at the exam. At an oral exam the task-text will never stand alone. The mathematical communication and questions asked to follow up upon the task – open or closed – is still the main focus of interest for what kind of problem the task will develop into and what kind of answers new questions will require.

1.2.1 Summary of Openness

The practice of open problem-solving in mathematics is characterised by cultural differences. Open-ended problem solving or the so-called 'open approach method' has been developed in Japan, the use of investigations was developed in England, and in the Netherlands the 'realistic mathematics' is approach seen as 'open problem solving'. Project work is a requirement in Denmark in lower secondary classes, and Skovsmose (2000) among others argues that project work should be considered for open problem solving in mathematics. Wistedt (2001) highlights the paradox about project work, and raises the issue of what to learn first.

Open tasks are used for assessment in California and Australia.

A categorical matrix involving only the initial text was presented above. The conclusion in all cases is that the mathematics communication following the open tasks is of crucial importance.

1.3 Communication

Communication, or more correctly mathematics communication, is highlighted as one of the main determinants for how open tasks inspire the pupils to work with mathematics. Communication is a process that needs one or several listeners as well as speakers. Luhmann (2000) describes communication as composed of three components: selection of information, selection of form, and selection of understanding. Inspired of this I will describe communication between two persons as a composition of four components:

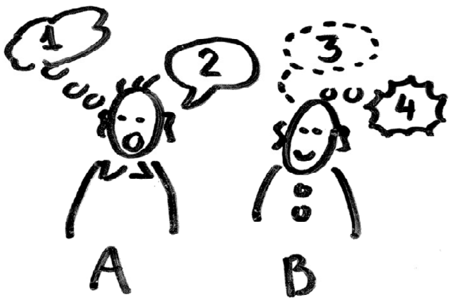


Figure 16: Communication between A and B: 1) selecting of form, 2) speaking the utterance, 3) listening to the utterance, 4) interpreting the utterance

As the figure shows, communication between two persons, will follow the four steps again and again. Communication could also be a monologue, where one person speaks to many people, as it often happens in the classroom. Still, listeners are needed and listening is different when an answer is expected than when the listener is only expected to listen without being required to make comments. The speaker understands the information (we presume) and selects the form, while the listener is left the hard task of understanding. If information and/or form are unclear, it may be difficult to listen and to understand what the speaker means. In this study, the role of communication in mathematics teaching revolves around how the mathematical communication is interpreted. How does the pupil interpret the teacher and other pupils, and how does the teacher understand utterances made by the pupil or a group of pupils? It is a question of listening. Covey's taxonomy (Covey, 1992) p. 240) contains five levels of listening: Ignoring the other, simply not listening at all; Pretending, uttering automatic words of confirmation but not really listening; Selective listening, paying attention to certain parts; Attentive listening, paying attention to the noticeable words, and maybe repeat the words of the person in order to check your understanding (active listening); empathetic listening is seeking an understanding of the person that goes beyond words.

Successful communication is crucial for what makes teaching work and, in combination with mathematical communication, it is what makes mathematics teaching work. Communication depends on the context of communication, and when the topic is open problem-solving in mathematics, the questions asked by either the teacher or the pupils are crucial for the mathematical work and understanding.

Wells (Wells, 1999) compares Vygotsky's psychological ideas of communication with Halliday's linguistic ideas of the same, and transfers their common ideas into practice. The main idea is the combination of language and social activity along with a process of making sense. In his classroom observations, Wells learned how inquiry was

> *'rewarding for a teacher who systematically investigates her or his own practice in an attempt to improve it' (p. xiii).*

Halliday highlights the reciprocal relationship that the way in which we interpret the context of situation largely determines what we say, and what we say plays a role in determining the situation (ibid, p. 10). Both Halliday and Vygotsky consider language to be the master cultural tool developed for and refined in service of social action and interaction. Both are interested in language as a means of development for the individual and for a particular community. The speech function is seen not only as a mediator for social action, but also as 'inner speech' which mediates the individual's mental activities of remembering, thinking and reasoning.

Dysthe (Dysthe, 1997) describes also how different kinds of communication can engender different kinds of learning. She distinguishes between using monologues or dialogues in the teaching:

- The 'monologue classroom' values the presentation, reproduction and evaluating of knowledge, where the knowledge is controlled and exists independently of the lesson.
- The 'dialogical classroom' consists of authentic and open questions, which the teacher uses to involve the pupils in the teaching situation to challenge their understanding and thinking and to help them to see the connection between what they learned earlier and the new topics.

Dysthe views authentic questions as questions without a fixed answer. When the teacher uses authentic questions, it is not to control knowledge, but to signal that the teacher is interested to hear about what the pupils have to say, their opinions, reflections or other things that the teacher does not know enough about.

'The triadic dialogue' (Lemke, 1990, here Wells, 1999, p. 126) or the IRF/E pattern, where 'I' is the initiate, 'R' is the student response and 'F/E' is the following-up comment or evaluation of the response, are crucial for the benefit of learning and show an underlying belief or cultural habit.

Alrø & Skovsmose (Alrø and Skovsmose, 2002) explicate an inquiry co-operation model (IC-Model) as a particular form of student-teacher interaction exploring a landscape of investigation. The elements of the IC-Model are:

> *…getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating – and they all exemplify dialogic acts. (ibid: 16)*

Teaching consists of communication and teaching mathematics through open practical problems is totally dependent on how the teacher chooses to communicate and on the way she encourages the pupils to communicate with each other. Teaching could always be done differently, and the choices about how to communicate the content could also have many expressions, but working with open problems calls for special techniques and willingness in the teachers to listen and to understand different solutions. It is therefore important for the teacher to be aware of her own communication – including listening – in the teaching situation.

1.3.1 Summary of communication

Communication is understood as articulated utterances and interpretation thereof; and maybe combined with an answer to the utterance, which again is an utterance. Teaching pupils mathematics consists of communication and mathematics communication is an equal part of this communication and teaching. Choices about how to communicate are crucial for the teaching. Communication influences the context situation, which again influences what we say.

The triadic dialogue is a common way to communicate in teaching, and so is the monologue, while the communication we look for when working with open problem solving is a dialogue similar to the IC-Model.

2. Before the course

2.1 The first interviews

Based on interviews made prior to C-02 with four of the enrolled teachers, I have listed their characteristics, expectations, beliefs, needs and aspiration, and linked the information to the way the course could benefit their teaching.

The interview was my first opportunity to meet the participating teachers on C-02. I went to their schools equipped with a tape recorder and an interview guide and met them on ‘their own ground’. The school buildings varied a lot, some very old while others were only a few decades old, but in all of them it was difficult to find a place where we could talk undisturbed – teachers do not have their own quiet places in the Danish schools. At one school, we conducted the interview in the ‘copying room’ where background noise from other teachers using the machine made it difficult for me to transcribe the interview afterwards. In another school, we had the teachers’ room to ourselves, because it was late in the afternoon. My aim with the interviews was to determine the teacher’s expectations of the course, and to understand their everyday teaching, which could help to interpret the data collected during and after the course. The interview setting was ‘asymmetrical participation’ (Linell, 1998), p. 221). I was the one to decide the topic. I wanted the theachers to tell me about their teaching, beliefs, successes and difficulties, and I was not a neutral person to them; I was their teacher educator-to-be, and they would soon be my ‘teachers’ . I was the listener, but I also influenced their answers with my comments and further questions. I presented the interview guide to them for approval before the interview, but made it clear that it was only to help us to remember what the main idea with the interview was; we could speak freely as well.

The semi-structured interviews guide they saw was formulated like the following; the first question deserves an explanation: to become a mathematics teacher is a choice that can be made on pre-service education, but also later on if the teacher so wants and the school needs it. Many mathematics teachers are not specialised in mathematics at all.

Why did you become a mathematics teacher?
If you think back to a successful mathematics lesson, what made it a success?
What makes a mathematics lesson unsuccessful?
What kinds of experiences have influenced you to change your mathematics teaching? How did they appear? Inspiration? Role models? Influences? Characteristic milestones?
How do you interpret the concepts open/closed tasks? Is using open tasks something you use consciously?
How do you expect the course to influence your daily work?

The oral questions were not formulated exactly like that during each interview, but they ran along those lines. For instance I would ask: “When you leave the classroom with a feeling of having held a good mathematics lesson, can you describe what happened and why it feels good?” The questions about inspiration sounds: “Are you aware of what inspiration you draw on when you teach mathematics?” I interpret the question about a successful lesson as their beliefs about what good teaching could be like, and for the question about the inspiration, I interpret their answers as their view of what inspired this belief. In the following, I have called the teachers Adrian, Tina, John, and Mona.

2.2 Presentation of the teachers

Adrian is a male teacher, specialised in mathematics, with five years of experience. He works at an old school in Copenhagen. To the question about 'inspiration', he responded:

> *One reason for me to be a mathematics teacher is the wonderful teacher educator I had, in my pre-service education. He influenced me with his alternative ways of doing everything. (…) I learned a lot from the Kongsted family (a Danish family that writes books about how to make workshops in primary school mathematics). (…) Mellin-Olsen says that if you want to change anything in your teaching, you should do it the first fourteen days, and so I did. (…) None of my colleagues inspire me.*

And, to the 'beliefs' question, he responded:

> *I want to work inductively, but the pupils feel so safe with the textbook, which is more deductive. It would be easier if they were raised in a different way. I like to see the children being active, that they learn something. And, I wish to find the energy to tell my colleagues what I feel teaching should be like; I am afraid to end like them, drinking my coffee and without ambitions. (…) The children have their own language and my tasks are in my language, which the pupils have to decipher before they can get to work and understand. Maybe it would be easier with common concepts and representation. The freedom to choose teaching methods is gone, but it is fine with me, it is necessary to know and be able to explain what you do. (…) The teacher should have the courage to go into the classroom and say to the pupils: 'Now we start from scratch and forget every thing you have learned, just forget it'. I did that.*

Tina is a female teacher specialised in mathematics with three years of experience. She works at a school in the countryside. To the 'inspiration' question, she responded:

> *The teacher educator I had as pre-service student is an idol to me, with her way of teaching mathematical interplay with mathematics in practice. Except for her, I don't have any role models, only my own awareness of what works, and I use trial and error.*

To the 'beliefs' question, she responded:

> *I look at myself as a specialist mathematics teacher. It is necessary to have professional qualifications, which give a general over-view of the teaching situation and make it more interesting; I feel the difference when I teach history, where I am not equally qualified. (…) I like the children to be aware and active, when they are in the learning process, but I miss some assistance when I try new methods. We can't always find answers to every question, even though the pupils in lower secondary think so; I want to break down the belief that there is always one true answer. It is different to teach both the little ones and the old pupils. (…) I believe that you will not learn to add unless you solve at least 1 000 tasks. (…) I thought I could do without a textbook, but I couldn't; yet, my faith in my ideology isn't broken, I still believe that there should be many ways to tackle tasks.*

John is a male teacher specialised as a mathematics teacher with two years of experience. He works at a town school outside Copenhagen. To the 'inspiration' question, he responded:

> *I really missed inspiration during my pre-service time. My role model is the big mister Keaton from the movie 'Dead Poets' Society'; I try to lean upon him. (…) My ideas and inspiration to think differently come from my leisure life as a scout leader. (…) I don't want to choose only one theoretical notion and be faithful to that. (…) I use some of the same methods as I was taught myself. (…) None of my colleagues have inspired me so far.*

And to the 'beliefs' question, he responded:

> *Mathematics is fun, that's why I became a mathematics teacher; it is fun to play with numbers and to think about what happens. Why do the children think the way they do? It attracts me, and I lack knowledge of these skills from my pre-service education. (…) It is important for me that the atmosphere creates enthusiasm. I like it when I feel that the pupils understand that we communicate about, if they know what's going on. The opposite is when I need to explain the same thing over and over again, and I can't come up with new ways of expressing it, then I hear my own voice repeating the same over and over*

again, it is awful. I like short introductions and then to go around helping the pupils. (…) I find that two pages with the same tasks is waste of time; if the pupils understand the first five tasks, they don't have to solve the next ten or twenty.

Mona is a female teacher with twenty years of experience, she is not specialised in mathematics and do not have any experiences with teaching mathematics in lower secondary classes. She was about to teach mathematics for the first time, which was her reason for enrolling the in-service course. She works at a school in Copenhagen. To the question about 'inspiration', she responded:

I take my inspiration from the textbook, which is fun, inviting and inspiring. (…) I haven't engaged in collaboration with my colleagues in mathematics. Earlier, there were more mathematics teachers at the school, but we didn't do anything to inspire each other. (…) I have never been on a mathematics in-service course before.

And to the 'beliefs' question, she responded:

I am not specialised in mathematics, but I like that it is a concrete subject; that is what I have to say about that. I like when all the pupils are active and learn something. I haven't had many such lessons this year, even though I use a lot of energy to run around and help the pupils. Many pupils with foreign ancestry in a class is the reason for the teacher to be very active all the time, and it is difficult to do it alone. (…) I teach them to use each other too. (…) I wonder why the transfer between situations is so poor; they are not able to use the things I teach them in mathematics in other practical subjects. Their skills are very situated.

Here is a brief overview of some of the significant reactions characteristics:
Adrian and Tina both mentioned their teacher educator from their pre-service training as their main role model. Adrian wanted to teach in different ways and do things in an alternative way like his in-service teacher did, and he wished that the pupils had been raised differently. What kind of changes Adrian wants, and by what means, we don't know. Tina felt her in-service education had prepared her well to be a professional mathematics teacher, and therefore she felt competent to develop her own practice alone through reflection. John mentioned that he lacked inspiration from his in-service framing, and that he turned to a fictional character from a movie and his leisure life as a scout leader for inspiration. Mona said that she never had any mathematical inspiration from her pre-service training and therefore she used the textbook 'as is'. None of them took any inspiration from their colleagues. It is important to know how the teachers find inspiration, because it will tell us where to focus. We don't know how Tina will develop her practice, and what John and Mona have learned from C-02 is provided below.

All reactors mentioned active pupils as essential for 'good teaching'. A learning process that involves active pupils has been a common place in the Danish Folkeskole for several decades, where requirements about learning 'about and through democracy' influenced the teaching practice (Korsgaard, 1999). The textbook still plays a major role for mathematics, as documented in a recent report (Danmarks Evalueringsinstitut, 2006) (The Danish Evaluation Institute), yet workshops in mathematics and group work are commonly used methods in mathematics teaching (Ejersbo and Nyholm, 2004). Focusing so much on teaching through activities has created the problem that teachers in their planning often choose activities rather than or before the content, which can cause a gap between what the pupils actually learn through the activities and what the teachers planned for them to learn.

The four teachers used three different textbooks, and because Danish textbooks differ greatly from one another, we know that the textbook influenced the teaching in different ways. The teachers referred to the textbook in very different ways during the interviews: one said that it was too deductive (Adrian used 'Cort & Johannesen'), one that she would be better off without it (Tina used 'Faktor') and one that it was very inspiring (Mona used 'Matematiktak') – John, too, used 'Matematiktak' and thought that it was a little mixed up, but he also had access to other textbooks. This can be interpreted as very telling of for how much the teachers depended on the textbook in their teaching. Mona depended on the textbook because it was her 'best helper', whereas Tina expressed a professional independence and that she wanted to use the textbook only when it suited her; in fact it seemed she would like to get rid of it, but she felt too young in her service to just discard it. Tina believed in doing the same tasks so many times that they become automatic, while John called the same practice a waste of time.

2.3 Practice-theory

These interviews show that these four teachers used their textbooks differently, and according to their beliefs, or what Handal and Lauvås (Handal and Lauvås, 2002) call 'practice-theory', the subjective theory the teacher has about her teaching, which is said, by the authors, to be the strongest influence on the teaching.. Practice-theory is defined as a part of the pedagogical practice, which is explained as consisting of three levels (P1-P3): The 'level of action' (P1), the 'level of reasoning' (P2), and the 'level of ethical justification' (P3) (Handal & Lauvås, 2002).

- The level of action (P1) illustrates what kinds of acting is going on when the teacher teaches in the classroom, which includes when the teacher explains things, asks questions, communicates etc.
- The level of reasoning (P2) refers to the teachers' explanation or reason for doing the kinds of actions; what the teacher answers to her own or others questions, why she chooses to teach like she does.
- The level of ethical justification (P3) means that when the teacher prepares or looks back at her teaching, she can justify her teaching as proper and defensible.

The practice-theory consists of the levels of reasoning and ethical justification combined. These two levels consist of experiences based on both theoretical and practical knowledge combined with values that justify the ethical norms. It means that practical expressions can lean upon three types of argumentation: Empirically based argumentation, theoretically based argumentation and ethical arguments (ibid, p.45).

Teachers' beliefs related to their teaching are investigated in several studies ((Schoenfeld, 1992,Pehkonen and Törner, 2004,Skott, 2000,Thompson, 1992)). Several more researchers agree that belief and teaching are closely related and interdependent of each other. The question is how the connection is realised in practice, and if and how the beliefs guide the practice or if and how the practice influences the beliefs. The teachers' daily work is closely related to their beliefs in their practice-theory. The concept practice-theory is explained as

Dale (Dale, 1989) describes three competence-levels (K1, K2 and K3), of teaching. Dale's competence-levels consist of: K1 how to conduct and carry out the teaching; K2 how to construct educational programs: preparing, organising and evaluating teaching related to the overall ministerial requirements, and K3 to communicate with and construct educational theory, which means to have a critical view on required programs and be able to explain and discuss it.

K1 is similar to P1, where in both explanations it is a question about action and how the teachers teach. K2 is relatively close to P2, but in Dale's categorisation the plans, reasons and argumentation for doing the actions are related to ministerial requirements, which means that the argumentation should be expressed in the same terms as used there.

K3 differs from the 'ethical' P3 in content; the ethical P3 level still concerns the classroom teaching, while K3 concerns knowing and dealing with didactical theories. Dale (ibid) states that reflection upon and knowledge about K1-3 through in-service training would improve the teaching.

The ministerial requirements are not mentioned in the definition of practice-theory, but it is implicit that the goal of the teaching on level P1 is based on these requirements. At the same time, a school can develop their own practice-theory, which is situated in very local conditions or habits. Our identity is created in the many social arenas in which we participate; in this relation the identity concerns being a mathematics teacher under these local conditions.

In this study I followed Adrian, Tina and John into their classrooms after the course, while one teachers Mona was interviewed two months after finishing C-02. The reason for only interviewing Mona was, as mentioned before, that she was very different from the three 'young' teachers, who were all specialised in mathematics. For all four of them I tried to find out how they acted on the teaching level and what their argumentation for doing what they did was, and from where they got their beliefs, among others the ethical or ministerial requirements. The four teachers expressed also their conception of mathematics and mathematics teaching in different ways, from 'concrete' (Mona) to 'fun' (John). John also expressed his interest in finding out how his pupils think, while Mona described her teaching as difficult to manage alone, because of the many immigrant pupils, which meant that she had to be very active. She complained that it was difficult for her to provide 'good teaching', and it seems that even though she wants her pupils to be active, she was the most active.

2.4 The teachers' expectations to C-02 and their use of open tasks

The interview guide was not followed slavishly; for instance it included the questions: "How do you expect the course to influence your daily teaching?", but during the actual interviews, my questions were not quite identical for each of the teachers. I therefore quote my own questions for each of the answers (L):

L: Why did you choose the course 'Mathematics for oral exam'?

Adrian: I have a group of pupils who are not ready for an exam of this kind. I need to know how the tasks for the exam should be formulated, and how the open tasks can contain more specific mathematics.

L: What do you expect to learn from this course?

Tina: I am desperate, my class is going to take the exam next summer and I feel so inexperienced. I hope to get ideas and tools for the day-to-day teaching where I can integrate communicative skills for the oral part of mathematics.

L: What kind of expectations do you have about this course?

John: I want to get some background knowledge, some ideas for the tasks and for how to carry out the exam - because I don't feel I have the necessary knowledge or skills from my pre-service education.

L: How do you expect the course to influence your daily teaching?

Mona: I hope the course will make me able to work with oral mathematics for the exam, to get some ready-made tasks to bring home and that I will learn how to create such tasks as well.

Tina and John expressed their feelings related to their missing competencies regarding the requirements of the regulations for the oral exam; while Adrian reported that his pupils were not ready for the exam, Mona seemed very pragmatic and described what she saw as necessary for her to conduct an oral exam. On the cognitive level, they expressed their needs for tasks, ideas, and tools for planning and for how to conduct this kind of oral examination. Expressions such as 'I am desperate', 'I don't feel I have the necessary knowledge or skills' show how strong their anxiety about this exam was. They expressed a need for help with formulating a problem, with how to carry out the exam and to find out what kind of communication to call oral mathematics.

In addition to the question in the interviews about expectations to C-02, I asked all the participants to write their expectation and to put the answers in an envelope on the first day of the course. I kept those envelopes during the course and gave them back in the end, asking the teachers to comment on their own expectations. The dominant response (19 out of 26) was along the lines of:

• "New ideas to make the tasks for the oral exam as required by the Ministry of Education" or "Improve my skills in creating tasks for the oral examination"

The second most frequent response (9 out of 26) concerned the daily teaching:

• "Inspiration for the communicative part of mathematics teaching"

Most of the teachers wanted to develop their skills at making their own tasks for the oral exam, as recommended official in the booklet from the ministry of education for the oral exam (1998). They wanted as well some ready-made tasks to take home. They also hoped for inspiration to improve their day-to-day teaching, but this was not as important to them as to manage the exam tasks. An oral exam is also a test for the teacher, since an external examiner controls that everything is of the required standards.

The concept 'open problems' was not defined in the first interview with the teachers, but they had all received the tasks I sent them, and some of them had already used them in their teaching. My question in the interview concerned their use of open problems in their daily teaching, and if they used open problems, I asked them how. Their answers for these questions were:

(Adrian started without any prodding from me.)

Adrian: When I try to work with open problem-solving, I feel I am unable to gather up the loose ends. The pupils end in place, I would never think of.

L: How do you define an open task?

Adrian: To me an open task is when the pupils should make some choices, but it is difficult to control an investigation with only one teacher and 26 pupils in a little room of 42 m2. I am afraid I could go to the extreme and say that now we work with open problems, I can relax; I have differentiated the teaching, and then let the pupils go where they can.

L: How can you break the pattern (that the pupils always want one exact answer)?

Tina: Through open tasks, where there are more different possibilities. I think of your tasks, which my pupils were happy with. They felt challenged and inspired to find more solving models. Some of the pupils came up with results I hadn't even thought about. Open-ended tasks give the pupils opportunities to work with their own ideas, but it is difficult to make open tasks that make them learn new skills.

L: What kind of methods do you use?

John: I teach more or less in the same way I was taught myself, and I don't want to be restrained of any certain theories. The closed tasks, I feel, are good for the weak or insecure pupils.

(…)
L: How do you think of tasks when I ask you about open problems?

John: Describe the relationship between Denmark and Malaysia through statistics. I see lots of benefits in closed tasks. (…) but perhaps there are some advantages to open problems after all; it is possible to bypass what you are not able to do.

L: Among the tasks I sent you, you saw some open tasks. Have you any experience working with open tasks?

Mona: From time to time I took some articles from the newspaper, for instance about fugitives in Denmark, where we worked with statistics. (…)

L: Have you thought about the way to ask questions, I mean an open way or a more closed way.

Mona: Yes, and what happens to the pupils, when I ask an open question? They get totally confused. "It is difficult to have an opinion all the time" they say.

L: How do you ask an open question?

Mona: I ask questions as 'How can it be that...', 'Why is it like that?' Or 'What does it means for the development?'

Adrian expressed difficulties with the loss of control when working with open problems, but has an interesting thought, when he said: "…I can relax; I have differentiated the teaching, and then let the pupils go where they can". What he says seems to be that he does not care where the pupils go, as long as he 'differentiated' the teaching, he did what he should. Tina was positive because she had a positive experience with the tasks I sent her, but still she doubted that the pupils would learn new skills by working with open tasks. John was not positive towards open problems before he saw the potential in leaving the difficulties unsolved. And Mona confused open questions with 'opinion questions', which confused her pupils. I interpret all of them to have a kind of 'homemade' understanding of what open mathematics tasks could be, and it would seem that none of them had much experience with working with open practical problems in mathematics.

For Adrian, John and Mona it seems as an open problem is closely related to project work. As mentioned above, it is required by law that project work must be mastered in the ninth grade. This kind of work has been used in the Danish Folkeskole for more than a decade, and many teachers have found a way to work with this requirement. Yet, mathematics is normally not involved, so one potential problem might be that they are familiar with open problems from other topics, but the transfer into mathematics tasks and mathematics investigation is difficult because of this limited knowledge about open problems.

2.5 Summary of the investigation before C-02

The teachers' expectations to C-02 showed that they felt pressured to learn how to formulate open practical problems and how to use such problems both in the test and in the day-to-day teaching. Furthermore, their statements about open problems in mathematics showed that their interpretation and their feelings concerned the application of open problems. The teachers' practice-theory has revealed through the interviews. The teachers' explanations of their way to understand and use open practical problems was based on their own practice-theory.

The data from the interviews will be used to investigate whether and how C-02 made a difference for each teacher; whether they felt better prepared and more competent in producing open problem tasks, and in using them in their daily teaching; and if and how their praxis-theory were influenced.

3. Preparing C-02

In this part, I will describe what governed my choices, in my preparation of C-02. I relied on the course prototype, my teaching experiences, theoretical ideas, and frameworks based on open practical problem-solving. Some of the descriptions are from my log; others concern the material and activities used for C-02.

Teaching planning consists of clarification of different elements such as:

Aim, content, varying teaching methods, materials, and methods to evaluate whether and to what extent the teaching was a success.

Preparation of C-02 took place in the period between the interviews and the actual course. Part of this preparation involved determining which workshops from the prototype course I should maintain more or less as they were, and which workshops I should change.

The prototype course and C-02 are described on p. 15 ff. In this chapter, I will concentrate on workshops that concerned: open problem-solving, communication, reflection and evaluation.

My approach to teaching was that the ‘teachers’ should be the active part: The one who works is the one who learn (Larsen, 1998). Accordingly, I designed C-02 to consist of small workshops with brief introductions and discussions together with situations that would give different input to the teachers. My intention was that it would be possible to discuss what kind of experiences the teachers got from this work, either in groups or in plenary discussions.

> *My plan is to make some warm-up exercises every day just after the lunch break, where we can try different kind of open tasks. I feel nervous, it is difficult for me as well, and I still feel uncertain when I practice. How can I evaluate and help the teacher the best way? I have to choose what to start with (in my planning), the content or the structure. I need to weave them together like a road weaves the numbers together through house numbers with odd numbers on the left side (for the content) and even on the right (for the structures). I wonder why it is so important for me to work with open problem-solving. I did it in my maths classes before I knew much about what I did and why. I still remember the special energy in the classroom. I want to share this with other teachers. (Log entry, 04.08.02)*

The activities planned for C-02 were developed from my own experiences as a teacher and teacher-educator combined with research ideas, as mentioned in the above description of the C-02. Applying research in C-02 did not mean that the teachers were given articles to read, but rather that I took some inspiration from research in the field.

At C-02, I used the theoretical framework ‘aesthetical learning processes’; a method I generated in co-operation with colleagues at the innovative centre ‘Statens Pædagogiske Forsøgscenter’ (SPF), where I worked for more than a decade. In this framework, Horh and Pedersen (Hohr and Pedersen, 1996) have developed a way to express what cannot easily be communicated verbally. The method is aimed to shape a space for expressing experiences that are still not completely formed in a linear way or completely obvious for the person involved. The idea is to create an emotional experience from the beginning, and from that platform to let the event be a personal experience, a need that precedes analysis.

‘Aesthetical learning processes’ can be depicted as a tripartite model for the process of experiencing a new conception:

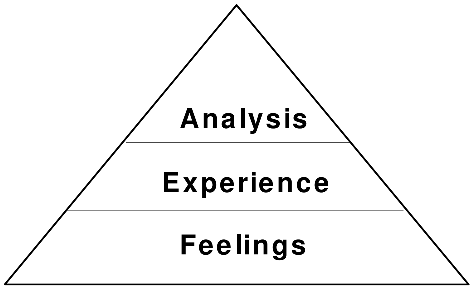


Figure 17: 'Aesthetical learning processes'

A conception arises at the moment feelings find a conscious form, which later becomes an experience that can be analysed. From the arts, where this method has its roots, the expression of the impression is again a 'non-linear' form without words such as a painting, a sculpture or a performance. In this study it was different: The expression had to end up in a linear analysis, expressed through words and with argumentation. Inspired by 'aesthetical learning processes', I used the part in which we perceived some common events in a kind of activity, which engaged and engendered emotions in a 'safe setting'; I will henceforth refer to this latter method as 'engaging learning processes'.

Using emotion as a teaching factor is mentioned elsewhere places in the research literature. Here I shall only mention Mason, who in his article (Mason, 1998) writes about the importance of awareness and emotion. He writes:

> *"Real possibilities (to teach effectively) emerge when awareness is educated and behaviour is trained in concert. And this is done by harnessing emotions. All this is summarised in:*
> *Only awareness is educable;*
> *Only behaviour is trainable;*
> *Only emotion is harnessable.*
> *(…) Teaching is then seen as a process of directing students in the harnessing of their emotions to provide the energy both to train their behaviour and to educate their awareness."*

This is in many ways similar to 'engaging learning processes' because emotion is harnessed to facilitate the teaching in both frameworks.

3.1 Planning of C-02, including materials

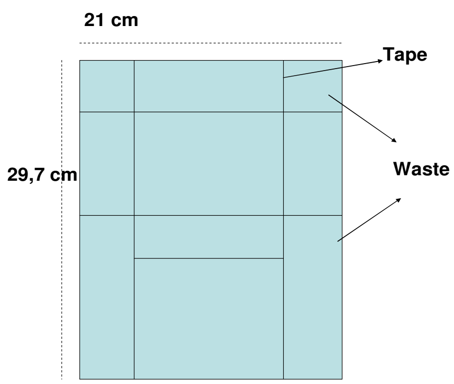
The main workshop in C-02 consisted of open practical problems. The aim was that the teachers should familiarise themselves with the official regulations and become able to produce and teach with open practical problems. The content consisted of the Ministerial requirements and different tasks to solve and reflect upon. Discussions were prepared about defining differences between open and closed problems. The teaching methods were mixed with group work, plenum discussion and plenum presentations. Materials were tasks I brought to the course as well as some of the teachers' own. For the first week I made a questionnaire that each teacher should fill out as evaluation, and in the end an official evaluation schedule would be used.

Following are two examples of the tasks in C-02. The first one is inspired by Clarke (Clarke, 1996):

Soap-packing
You have a task that must be completed in two work weeks.
You will do the work at home. The job is to pack small bars of soap in boxes that will go to different hotels around the country. You will receive DKK 4 000 for the job, but you have to pay the transportation costs yourself.
You pick up the material on Monday, 4 July. First, you take the bus to the soap factory. Then you take a taxi home with the soap and the boxes. The bus trip costs DKK 20. And the taxi ride costs DKK 180.
After one week, you realise that you cannot complete the job by Friday, 15 July. You call Anna and ask her to help you. She can work Tuesday, Wednesday, and Thursday - about 30 hours all told.
When you begin working together, you discover that Anna work faster than you do. Anna packs an average of 150 soaps per hour and you pack 100 soaps per hour.
You finish late Thursday night. You and Anna take a taxi on Friday morning to drop off the soap boxes. The taxi ride costs DKK 200. You take the bus home and it costs DKK per person 20.
Two weeks later, you get paid DKK 4 000. How will you split the money between the two of you?
Make at least two different budgets that will show how the money can be split. Explain why you have split the money as you have.
Present the solutions on OHP transparency. The group has twenty minutes to solve the task.

The other task was developed together with P. Valero, who was a Ph.D. student at DPU at the time:

The chocolate box
The owner of a factory wishes to produce a box for chocolate. He wants a box, which can be produced from a single piece of cardboard that measures 21.0 cm x 29.7 cm. He wants this design:



He asks his consultants to design the box with the following requirements:

1. Waste as little cardboard as possible
2. Use tape for the corners, but as little as possible (Notice: Cardboard and tape are both very expensive)
3. The volume must be maximised

The problem is: How should the box be designed to satisfy the factory owner?

Solving the tasks should be group work. The soap task was meant as an example of an open practical tasks and the presentation on an OH transparent was practicable to them as well in their own classrooms. A further requirement, when solving the 'Chocolate box' task, was that each group had to find a person in the group to serve as observer of the process when the group solved the task; the observer was not allowed to speak during the task. The total allotted was 40 minutes, and the available material was paper, a pair of scissors, cardboard and tape. The plan was for each observer to report from the work afterward. The 'Chocolate box' task was prepared as a role-play to practice observation of how the participants solved the task in the classroom. The tasks were meant to develop participants' awareness of how 'pupils' solve a task, how they communicate or how they use mathematics; the observer was the one to decide what to look for.

3.1.1 Other tasks

Different other tasks were prepared for the teachers:

One task, ‘Draw a bird’ had them to ‘practice constructivism’, to demonstrate that we have different interpretation of the same word. The introduction to this task was taken from a drawing book for children (Brookes, 1988), where seven different steps to draw were demonstrated. The explanation was illustrated by pictures, but the participants just heard the instructions, which I read slowly one at a time without showing the pictures. After the seven steps they ‘ought’ to have the same bird, but of course they did not.

Another task, ‘Make a pattern’, had them create patterns with coloured pieces on their tables only by listening to each others’ explanation of a pattern the speaker held hidden.

The tasks were conducted so that I would stop them after a short period and ask the teachers to reflect upon what happened: What kinds of words were invented for the occasion and where did the language of mathematics help them?

Another role-play was planned for how to run the oral exam. The practical problems used should be the tasks that the participating’ teachers’ completed the second day. The ‘teachers’ would be split into groups, where each group had to provide an ‘examining teacher’ and an ‘external examiner’; the rest of the’ teachers’ played pupils, who should solve a task in pairs. The picture (p. 17) shows three ‘teachers’ who work as ‘pupils’. They worked with an open practical problem directed by an ‘examiner’ and an ‘external examiner’. This was scheduled to go on for an hour. Then the ‘examiner’ and the ‘external examiner’ openly evaluated the piece of work; the ‘pupils’ were allowed to listen, but not to comment. The last step in this workshop was that the ‘pupils’ evaluated the situation and how the ‘examiner’ and the ‘external examiner’ asked questions, the formulation of the task etc. This was meant to be the reflecting part. Then the ‘teachers’ should meet in the ‘task-design groups’ to redesign their own task, taking the evaluation into account. This was a way the ‘teachers’ could benefit from the role-playing workshop. No time was set aside for discussing their reflections in plenum.

The last two days of the first was set aside the ‘teachers’ to design a course for their own pupils. The teachers’ courses in their own classes were to be carried out before we met again ten weeks later. Reactions and reflections from practicing this in classes would serve as background for the last session of C-02 aside from some practical tasks for reflections. The aim was to practice a way for the ‘teachers’ to examine their own practice and reflect on it together. The content was their logs and questions, formulated for the work with the logs. These questions should make the ‘teachers’ think critically on the log reflections and make some questions as they read the logs, then categorise the wondering, and at last reflect the whole thing. The teaching methods were mainly group work with presentations in plenary. Their reflection should primarily take place in the groups.

3.2 Reflections on the planning

I organised the teaching methods and the group work, which was based more on experience than on any research about teaching. The methods were, as mentioned before, chosen because of my conviction that emotions play an important role according to learning and that the ‘teachers’ should be activated through different activities. The research mentioned in the program concerned to the content of C-02 rather than the teaching methods used on the course.

At the time I prepared C-02, I was mostly influenced by my own ideas about how open problem-solving could be transposed into effective teaching; and I was not really aware of where all my ideas came from. Therefore, my hypothetical learning trajectories were that if the ‘teachers’ were active working with solving open tasks, formulating and evaluating them, they should after the course be able to:

- Produce open mathematics problems
- Encourage their pupils through mathematical communication
- Be reflective in action as well as reflective on action

Maybe this was naïve and not exactly ‘reflected’, but with the good evaluations I used to get, I felt that C-02 would work as intended.

3.3 Summary of preparing C-02

The preparation concerned both teaching methods used at the course and the content for the course. The teaching methods build mostly on activities based on ‘engaging learning processes’ and experiences from many years of teaching. The content was based on theoretical frameworks about practical and open problems described in the research literature and on my own experiences with open problem-solving in my own teaching.

My hypothetical learning trajectories were that the’ teachers’ should after the course be able to:

- Produce open mathematics problems
- Encourage their pupils through mathematical communication
- Be reflective in action as well as reflective on action

4. During the course

I shall only describe a few details from the workshops: open problem-solving, communication, reflection and evaluation.

For the workshop about open problem solving, I used a revised version of Skovsmose's 'Milieus of learning' matrix for several years on the in-service courses, and it seemed to work. I worked together with Professor Ole Skovsmose at the time he developed his model and discussed it with him, and from his idea, I developed my own model to use on in-service courses.

Normally, when I used it, I gave the teachers examples of tasks for each category (Ejersbo and Mogensen, 2001); however at C-02, I used it differently. This time, I asked the teachers to come up with ten tasks each, write them on post-its and categorise them in a table that I drew on the blackboard. The picture shows both the table and the post-its.

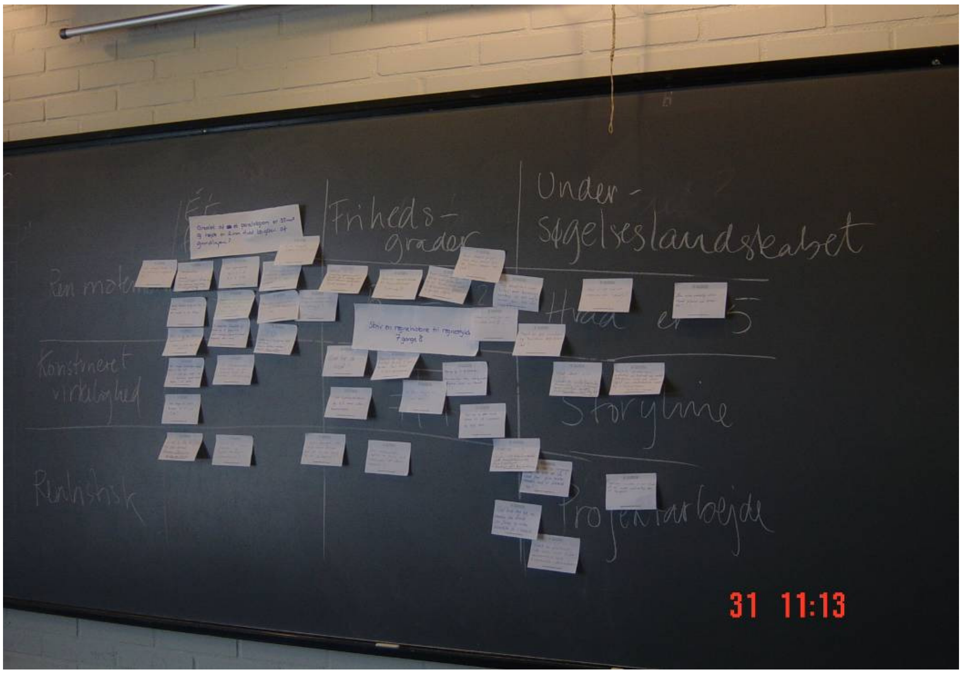


Figure 18: Picture of the matrix with the landscape of investigation used on C-02

This turned out to be difficult, and led to an interesting discussion about categorisation. It was much more difficult to categorise tasks than to find some tasks that fit into the table, because the matrix is not a categorical differentiation. My hypothetical learning trajectory failed. I had to change my plans and what I hoped for turned out to be impossible. I used this model with success many times, because I had prepared the tasks to fit into the categorisation, but when I turned it around, I could not control the process in the same way – and it failed. Together with the teachers, I learned from this mistake that if a system is not differentiated categorically, then it is impossible to categorise the individual tasks exactly into the system. We concluded that the task itself is only the beginning; the teacher's beliefs and pedagogical content knowledge is what guide the situation when pupils solve tasks, and how open or closed a task would be depends, as Watson and Mason (1998) pointed out, on the teachers' questions.

4.1 The two tasks: 'Soap' and 'Chocolate'

The two tasks were used as inspiration for the teachers' own planning of the course they would give their own pupils.

The instruction for the soap task was:

1. Spend twenty minutes in groups to find solutions and write them on one OHT
2. Groups present their solutions

The 'teachers' were all very active in the process, but in the presentation, where everybody could ask questions, I was the one who asked most of the questions. I had planned that the teachers should then make other tasks inspired by the soap task. They made some suggestions, but no real tasks and the ideas they came up with for new tasks resembling the soap task were: Make a drawing to describe how two classes can share a classroom together, ask questions based on a fairytale, about transportation from one place to another or cross country running, transport of oil, production of milk and so on. These suggestions were very imprecise and only half-baked ideas that the reactors could use in their planning of their own courses, which took place in groups. After the presentation, we left that particular task, so the teachers could use it as they liked.

The chocolate task was organised differently: Each group chose a member to play 'observer'. None of the teachers were happy to be the observer; they preferred to solve the task. The observer was not allowed to speak for the forty minutes the group worked on the solution of the task, but the observer was to summarise the process afterwards. The observer had to observe whatever she found interesting in the solving process. Before the presentation, the group had five minutes to listen and ask questions to their own observer. Some groups just made a box out of the paper and found forty minutes too long, while other groups only did calculations and some groups did both; the latter two found the time suitable. Again, little was asked at the observer's presentation and I finished this session by showing some different solutions to the task. I left the reflection to the teachers themselves.

The work with planning their own course went on in groups, while I worked as a consultant to the groups. In Mona's group, we had a discussion about how to describe things in a mathematical way. I asked Mona how she would describe a Coca Cola bottle, which stood on the table. She said:

"It is a container for Coca Cola and it is curved."

"Can you say more about it?" I asked.

"It is made of glass; I don't think I can say more about it"

"Is any of you able to add to the description?" I asked

"The height is about 16 cm, it has two circles of different diameters in the bottom and at the top, the pattern makes curves which again change the diameter and circumferences, the curves could be described in some way, the bottle itself has a mass, and the volume is 25 cm^3, then we can find the weight of the Coca Cola." others from the group commented. Their final task looked like this:

Cola!
Cola is today an essential part of youth's life. "Cola" exist in many different disguises, which means that cola is produced by many different breweries and comes in many different types of packaging and at many different prices.
This case is what we will work on from different problem perspectives.
Common for all:
1) Consumption among pupils:
* Produce a questionnaire
* Use statistical analysis on the collected data
* Produce a diagram that shows the results

2) The packaging:
* Perspective drawings – a can, a box
* Scale – draw a box to scale with space for the bottles
* Design your own soft drink bottle - create a logo for it
* Garbage – use your findings about the consumption of Cola, to find the weight of the waste in kg
* Quantitative description - geometry, colour, sizes, etc.

3) Prices:
* Describe and evaluate at least 4 different Cola types related to price and quality
* Discuss what determines the price of a Cola
* Is Cola on sale this week?

After this task, follow three other tasks to choose for group work, they are all about Cola in different contexts: One is about preparing a party for the class where Cola will be served; another is about a kiosk-owner, who drives to Germany and buys Cola; the third one is to prepare and organise a party for a whole lower secondary school. (For the whole text (in Danish) see appendix III)

In the same group, one of the teachers had a task about a chicken farmer, which she showed to the others. The task contained some six pieces of information about chickens and some tasks, where the pupils had to choose relevant data for the answers. One piece of information was that the farmer had twenty metres of fence to make a chicken fencing, and the task was to find out how large an area he can fence in – and how small? Furthermore, there were questions about how much he could earn per chicken.

Other groups chose a task about how to build a summer cottage; others planned how the class could save up money for a trip to Holland and used computation of compound interest; others planned to use football as basis for further calculations. My role in this part was to talk with the teachers about their ideas and ask questions that hopefully could inspire new ideas. Every teacher had a copy of all the tasks from this work, which gave rise to the planning of nine different courses containing ideas, lists of materials, aims, ways of organisation and how to evaluate the different courses.

4.2 Communication in use

Solving the task 'Make a pattern', where the teachers in pair should explain a pattern to each other, made them laugh and realise how difficult it was to be precise when they wanted to formulate how the others should re-create the pattern. Again, I left it to the teachers to reflect upon the task.

The role-play was a combination of communication and evaluation. Role-plays are not always a success with teachers as players, but the way this role-play was presented made it acceptable.

The teachers were in a sense to play themselves, but under other conditions than they were used to. All of then had to do the best they could: the 'examiner', the 'external examiner' and the 'pupils'. The game was organised so that the tasks produced were evaluated through the communication in the role-play. The teachers who played pupils in this game evaluated both the task and the way the examiner asked questions, while the examiner and the external examiner decided the rules. Their final discussion was to make them reflect upon both. This discussion was made in the groups only.

4.3 Reflection in work

I only received six teacher logs, out of a possible 27, with thoughts about their own courses taught in their classes. When we met again, I started with group work in the old 'course planning groups'. In each group, the 'teachers' had to describe and reflect upon their own courses, choose one of the instructional sequences taught in one of the classes, and present it to the others. The questions to reflect upon were:

- What went according to the plan? What happened that you couldn't foresee?
- Did the pupils learn what you expected?
- What worked well in the plan? Why?
- What was the most important feature? In what way?

Each presentation should last for ten minutes; again, there weren't many questions except for mine.

The logs I received served as a point of departure for another task. The logs were anonymous and copied with the following questions:

1. Use ten minutes to read the pages and write at least three questions that came to your mind when reading this.
2. Present the questions to the group and explain why they are relevant. Use five minutes for each.
3. Categorise the questions.
4. Prepare a presentation including the way the questions are categorised and reflection upon the categorisation, and choose a single point for further qualification.

The presentation was again limited to be on only one transparency, which should be a 'discussion paper' for each group. Unfortunately it turned out that the teachers found it very difficult to come up with three questions, and they could not use it for reflection as planned. Instead they evaluated the teachers who wrote the logs. My idea for this workshop was that their questions would serve as a basis for reflection, but this plan did not succeed. The discussion did not come close to the reflection I had imagined; instead some of the teachers were mean to a young teacher, who had sent her log showing some of her problems. Even though the logs were anonymous, she told the others which one she had written.

4.4 Evaluation

The evaluation of C-02 comprised several steps. After the first week, I made a questionnaire (a completed copy can be found in appendix IX) where the 'teachers' could express how they experienced the different parts. This resulted in the following general answers:

- It was fine to be informed about the Act, but a little frustrating too because of the many demands.

- ‘The good questions’-discussion was both relevant and made us reflect on our preconceptions.
- The part with the IT-programs was not interesting because of the way it was presented.
- The production of the open practical problems was a good idea.
- It was interesting to be in an ‘exam situation’ even though it was difficult to commit oneself to the game.
- C-02 provided good inspiration for how to produce a course in our own classes.
- The communication tasks were fun.
- The production of the course for our classes was difficult, and gave us all stress, but it was effective.
- C-02 had many good tasks and pieces of information, but maybe it was a little too much.

Teachers are normally not afraid to be honest; they give praise or the opposite freely. They were pleased that they returned to their schools with not only ideas, but a planned course for their own classes. Most of them conducted the planned course in the intermediate period. Most of them failed to write a log and send it to me.

Another ‘homemade’ evaluation was the paper the ‘teachers’ wrote in the beginning about their expectations. I gave each of them their envelope back and asked them to evaluate C-02 compared with their early expectations. Again, most of them were positive, and reported that they got ‘something’ to take home with them. A few of them said that they did not understand why C-02 focused so much on the day-to-day teaching rather than on the exam. They found that they did not want to change their teaching, but only wanted to know how to conduct the exam. I only read the written evaluation after the course and we did not discuss it.

The final evaluation questionnaire was an official form from the institution that held the course. The overall evaluation was, again, very positive; the teachers reported that it had been inspiring to work with other mathematics teachers and to be encouraged to produce so much. The following short summaries are from the completed questionnaires from the ‘teachers’ I interviewed before C-02 (Adrian, Tina, John and Mona); I am not quite sure about Mona’s evaluation, because her name was not on any of the completed questionnaires. I guess that the one without a name is hers, and that is the one I bring here.

Adrian: Very inspiring, and with a good organisation and structure. The course will change my teaching. I will try to inspire my colleagues too. It has been a good course, but the older teachers can be difficult to deal with, because of their view of teaching, of human nature etc.

Tina: My expectations about new ideas and material that I could use in my class have been met. I already use my new knowledge in my day-to-day teaching and I share with my colleagues too. I liked it that we had the break where we could try out the planned teaching and exchange experiences when we came back.

John: My starting point was no knowledge, and now I feel much more ready to conduct the exam. I would like to try to be more open in my teaching. I liked it that we had so much work with other teachers where we could exchange ideas. I liked the good spirit and attitudes in the big group.

Mona: I was satisfied with the course, but will not use it in my daily teaching, only in the design of the exam questions. I liked to talk with other colleagues, what I missed was to work more with the exam questions.

Only a few responded that the emphasis should be more on the exam tasks and less on the daily work. Only few wrote a log, but nearly all of them ran the course they designed during C-02 in their classes. A question is whether they got what they came for, and how it has influenced their teaching with open problem solving in mathematics.

4.5 Summary of C-02 practiced

Not all the workshops went as planned. The introduction to and clarification of open problem-solving was not clear enough at the outset, and furthermore it did not help to let the teachers believe that Skovsmose's matrix was categorical. The rest of the tasks followed the hypothetical learning trajectories with only small adjustments 'on the fly'. At the time we finished the course, I was not aware what benefits each teacher took home with him or her. Based on the evaluation and the teachers' expression of satisfaction, I expected naively maybe that they were prepared to teach with open practical problems in such a way that their pupils would benefit from it.

5. After the course

The next step was a closer inspection of how the exercises from C-02 were transformed into teaching in four classrooms. I planned to visit Adrian, Tina and John for video observations and I obtained permission from the pupils' parents as well. Mona's profile was quite different from those of the other three: she had many years of teaching experience, but this was the first time she taught mathematics in lower secondary classes, and she was not specialised in mathematics. Therefore I decided just to interview Mona one more time at her school. This interview was carried out a month after C-02 was finished.

I found that teaching mathematics through open problem-solving lead to more troubles than I had expected. In the case studies, I looked for different kinds of mathematics communication and I particularly looked for problems that the teachers ran into in their teaching with open problem-solving. In the final part of the pre-study, I investigated how I had taught on C-02, in particular the parts which should help the teachers not to have more troubles than necessary.

5.1 Case Study 1

As Case Study 1, I conducted a follow-up interview with Mona.

I wanted to know about Mona's 'practice-theory' and how it was put into practice after C-02. I met her at her school on an early morning before her own teaching began. We talked for nearly an hour. The interview guide was based on the first interview, and Mona saw this guide before we began the interview:

Based on C-02, I would like to hear what you actually do now when you teach mathematics.
Can you tell me about any good lessons you have hadsince I was here the last time?
Can you tell me some details from that particular lesson?
In what way to do you see mathematics as a concrete topic?
What do you like about mathematics and about teaching mathematics?
In what way is your textbook fun to work with? How do you choose materials from the textbook, which criteria do you value?
If you had more time, what would you do differently?
Methods: How do you see yourself as a mathematics teacher? What do the pupils learn? How? How do you keep them 'awake' now? How will you reach your goals? How will you help pupils who experience maths as a waste of time? How can you help the pupils learn more? How will you motivate them to learn more mathematics?
Did you miss having a group of subject teachers to collaborate with? (In the first interview Mona reported that she did not have any colleagues to collaborate with at school who were specialised in mathematics)
Have you changed anything in your teaching since the course? What is your view of open problems now?
Did the course meet your expectations? Which? How? Why?

The guide was only used for inspiration during the interview. A transcription of the interview is shown below in two columns with the interview in the left column and comments in the right. I used these signs in the transcription: 'L' for me, (…) means a 'jump' in the text, while … means a pause, and (?) means that it is inaudible:

Speaking	Commentary
[1] L: Did you have any lessons lately you were satisfied with? I mean, since you left the course? [2] Mona: Yes I did, but I don't think it had anything to do with the course. [3] L: No, but…	
[4] Mona: Yet, we talked on the course about compound interest in many different ways and we learned about the computer program to handle such data. [5] L: Yes	I was a little surprised that she said that it had nothing to do with the course and then immediately referred to the course, but on later inspection, it became obvious that she refers to a lesson which satisfied her expectations.
[6] Mona: And we had a lesson the other day, where we talked about it.	
[7] L: Yes [8] Mona: And it was, so to say, a lesson early in the day, and I felt that they were all very engaged.	I felt a tense atmosphere at the talk and decided to let her tell about the lesson only encouraging her with a 'yes'.
[9] L: Yes [10] Mona: And I will say it went straight ahead home. (…) [15] L: How did you start that lesson? [16] Mona: I wrote a situation on the blackboard about putting money in the bank and what could happen. (…) (…) [47] L: Is it something you have from the text book? [48] Mona: No, not necessary [49] L: Is it something you chose to take in?	
[50] Mona: It is something we talked about, it comes from the pupils as well (…) I ask them questions about their own experiences. (…) (…)	In the first interview Mona told me about her dependence on the textbook, so I asked her where she got the idea she used.
[73] L: Now, when you succeeded with such a lesson, do you feel it concerns mathematics communication? I remember you talked about that before the course. [74] Mona: It depends on how broad it can be. I still have doubts about how precise the mathematical language should be. (…)	I am interested in finding out about her practice-theory of what she thought mathematics communication consists of. In fact I know that mathematics communication consists of both 'form' and content knowledge concerning mathematics. It seemed she was still uncertain of what oral mathematics was, what to expect, how precise the language should be etc. I asked her, because I knew that in her group we discussed that as well when we talked about the Coca Cola bottle.
[91] Mona: And how can I make it easy for them to understand, let us say reduction (…) [92] L: What do you think of as the most important when you teach reduction in mathematics?	
[93] Mona: Yeah, it is the simplification, isn't it? My mathematical overview isn't so good that I can say, this is necessary and this is not. (...) I do it fast; it is only for the test. I can't send them to the test without…	She was the one who brought in reduction and I was curious to know her learning goals for a topic as reduction.
(…) [111] Mona: At the course, I got a task about chickens with information on one page and	Again I felt her concerns. It was difficult to make any comments to the fact that she did not see or know the reason or necessity for the topics she needed to teach. My role was not to teach her, but to

suggestions on what could be calculated on another page. I brought this to my class to practice oral mathematics. They also had the one from the course about Coca Cola, which they liked very much and found exciting, but the one with chickens, they didn’t show it any interest. The content seems to be important. (…)	make her talk about her teaching. Therefore I could silently listen. In the beginning of interview [2], I understood that her good experiences in her teaching were not influenced by the course, but here she mentioned several things, she used from the course and mentions that the pupils liked the task very much and found it exciting.
[118] L: (…) I am curious to know whether you have changed your beliefs about open problems solving. [119] Mona: No, what can I say… I still need to take care of the pupils in the class. [120] L: Yes [121] Mona: Some of them would… just go straight through all the points, and if the last three were open, they would make them as well; but some others would only try to solve the first four bullets. Therefore, it is important that it is very structured. I will not call them closed, but there should be some questions that aren’t so difficult. And all the tasks should be built in that way.	The task about the chickens was brought to the course for the little group Mona was a part of, and I never saw it. What I know about the chicken task is from Mona’s explanation; it contains information about a chicken farmer and his work combined with questions like how big a chicken yard could be, and his profits from eggs. I was curious to see if she changed her understanding of open problems or questions. She said that her pupils were confused when she used open problems, but she mentioned two tasks from the course, which she used. The task she described about the chickens was indeed an open-ended task.
[122] L: Can you explain a little more about the structure, which is neither open nor closed, but helps the pupils? [123] Mona: Let us take the one with the chickens. It was good because it had many opportunities, but it was still simple, even though it was referring to a complex reality. You have twenty meter of fence to make the biggest yard possible. Some of the pupils will make a lot of calculations from that information and those questions. (…) Other would like to find out how much you can earn on each chicken a year, with graphical drawings as well. (…)	The structure she mentions, reminded me of the way mathematics teachers often talk about their ‘weak mathematics pupils’, that they need to be guided through a task. Therefore I want to know more about it. I asked for the structure, but the answer is more about the way she sees the possibilities to use its openness.
[132] L: If you work with the open concept, which can have many degrees of freedom, as you said, and you want to take the pupils’ knowledge and interest into account, which plays a role as you said, how would you do it then?	
[133] Mona: They don’t have any. Some of them don’t have any knowledge. That’s the case. (…) [134] L: How do you interpret the word knowledge? [135] Mona: … understanding… I will… something about being a part of society … where numbers are treated and salaries are calculated… it is the way society is organised.	Here it seems that we do not share an understanding of the word ‘knowledge’. In [50] Mona talked about what came from the pupils as well, but when I asked her about the same, she refused that they contributed anything.
[136] L: like information about being a citizen in Denmark? [137] Mona: Yes (…)	It turned out that she meant that they did not have ‘knowledge about Danish society’ with its rules and regulations.
[189] L: Are you using any open questions to coax	Again I had the possibility to investigate whether

a reply? [190] Mona: Yes, for some of the pupils. [191] L: But you don't do it to all of them? [192] Mona: No. [193] L: What is so dangerous? Does it confuse them or what? [194] Mona: Yes, they get paralysed. If they just understand something, I should not confuse them with asking a question that totally crosses the line. [195] L: But the question doesn't have to cross the line; it could be just an open question concerning what they are doing. [196] Mona: Yes… of course (…) the problem is that I see the purpose of the open questions to be for the capable pupils, so they don't fall asleep.	her beliefs about open questions had changed. She showed emotions about the question and that was why I provoked her a little by asking about the 'dangerous'. But she repeated the same beliefs as in [121]: Open problem solving is not for pupils who have difficulties, only the for capable – and only so that they don't fall asleep. The open questions seemed to paralyse some of the pupils. Her understanding of open questions was apparently more an understanding of open questions in the social context the tasks were imbedded in [136]. Still, in her narrative about her teaching, it seemed that she used open tasks more [123]. Yet, it is difficult to know what the teaching was like without visiting her classroom.

I must admit my own tentative feelings, when I interpreted this interview. It felt like I was Mona's teacher more than a researcher. Mona's most important tool for inspiration was still the textbook, as she had told me in the first interview. Her mathematical overview was not exactly excellent [93]. At C-02, she met other mathematics teachers and found inspiration, which she later used [111], but it seemed difficult for her to identify exactly how this inspiration influenced her practice-theory. I cannot evaluate her teaching without having observed it, but judging from the interviews, I recognised her problems with teaching mathematics through open problems; her interpretation of open problems came from topics other than mathematics [135]. She has never learned about mathematics or how to use open problem solving in mathematics, however she was an experienced teacher in other topics and familiar with project work..

I got the feeling that she did not want to say that the course inspired her [2]. In the interview, she mentioned quite a few examples that showed her inspiration [111] [123]. It is not difficult to see or hear if a teacher lacks the mathematical skills they need for teaching. In her case, she even admitted it [93]. I felt that she was afraid that I would judge her, and maybe I did.

Her practice-theory was that the clueless pupils needed a rigid structure [121], but she was unable to explain it more clearly than 'the tasks should be closed'. In my experience, this is a fallacy; maybe the pupils are clueless because they never were allowed to do interesting investigations guided effectively by a competent mathematics teacher. Her beliefs about using open problems in her mathematics teaching seem to be influenced by the open problems, which are often used in project work. In the first interview she said that the pupils were very confused when she asked any open questions, but it seemed as if her understanding of open problems has changed with regard to some of the pupils. The 'chicken task' was open and she liked it, even though her pupils liked the 'Cola task' better.

I believe Mona is no different than other teachers in a similar situation. She was asked or permitted to teach lower secondary classes in mathematics, she got a short in-service course on how to fulfil the oral exam requirements, and then she was 'prepared'. However, in my opinion she was not, and I felt she was a victim, and aware of it, when she was confronted with the mathematics topics that she taught. She needed to defend her own position, even though I did not feel I attacked her. Perhaps the situation threatens her. My questions, when I asked her about her practice-theory, seemed to provoke her and made her defend her teaching.

In his article, Shulman consider (Shulman, 1987) what, for a teacher is the minimum required knowledge, including pedagogical content knowledge. Pedagogical content knowledge he describes as ‘that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding’. Pedagogical content knowledge has some similarities with the German ‘Stoffdidaktik’, according to which the primary task of mathematics education is to prepare mathematics for students, or to simplify in a methodical way, the mathematics knowledge that is offered by mathematics as a scientific discipline (Steinbring, 1998). Shulman (1987) reproduces (from Grossman, 1985) a story of a teacher who is observed in several teaching situations with the same class, both when she teaches in an area she knows a lot about and when teaching in areas where she is uncertain. The teacher’s behaviour and teaching differed a lot. In the first case the teacher’s teaching style was student-centred, discussion-based, occasionally Socratic, and otherwise interactive. When the teacher was uncertain, her teaching style was a highly didactical, teacher-directed, swiftly-paced combination of lecture and tightly-controlled recitation. After the lesson, the teacher admitted that she had actively avoided eye contact with students, so they were not encouraged to ask questions. This example illustrates the way in which teaching behaviour depends on comprehension. Shulman (ibid) claims that the flexible and interactive teaching techniques the teacher used were simply not available to her when she did not understand the topic to be taught. As I understand Mona’s story, she did not have a professional understanding of the subject, she only had the textbook, and therefore open-ended problems techniques were not available to her in a form where she had to produce them herself or to use them in an effective way.

Summing up Case 1, I conclude that Mona found some input to renew her teaching on C-02. She met other mathematics teachers and learned what it was like to exchange ideas and discuss the didactics of mathematics with other mathematics colleagues. The ‘Skovsmose model’, which I used to explain the difference between open and closed problems, could even have pushed her further in the direction that working with open problem solving in mathematics and project work were the same. My conclusion is that the benefit of using open problem solving in mathematics could not be adapted into her teaching in a reflective way at this stage, at least not in the way it was taught – or intended to be taught – on C-02.

5.2 Classroom observation

I visited, observed, and videotaped three teachers, Adrian, Tina and John. They were treated and interpreted as three case studies. In my observations, I focused on the different types of mathematical communication that appeared in the lesson I visited. I turn to give an overview that shows the framework for their teaching, before describing them individually.

Teachers / Activities	1: A man from a school in Copenhagen. Classroom size: 45 m2 (Adrian)	2: A woman from a school in countryside. Classroom size: 2x45m2 (Tina)	3: A man from a town school outside Copenhagen. Classroom size: 75 m2 (John)
Time table (lesson)	8:00-9:30 90 minutes Grade eight: Sixteen pupils	12:00-12.45 45 minutes Grade nine: Eighteen pupils	10:40-11:50 70 minutes Grade eight: Twenty two pupils
Presentation of the work the pupils had to work with in this lesson	Six minutes: Introduction to a new mathematics topic	One minute: The lesson was number 3 out of 4 consecutive lessons and the pupils knew what to do in this lesson	Five minutes: Introduction to a new organisation of the lesson
Group work	Thirty-five minutes: Groups were set up and given tasks by the teacher	Forty-three minutes: The groups continued their work from the previous lessons with tasks given earlier	Fifty-five minutes: Groups were set up and given tasks by the teacher
Plenary discussion	Forty minutes: Tasks and solutions were discussed	None	None
Second group work	Ten minutes: New tasks were given to be dealt with in the groups	None	None
Ending the lesson	Thirty seconds: “What you haven’t done here should be finished at home”	Twenty seconds: “We continue next lesson, which is the last time”	Three minutes: A short summary of the lesson and the comment: “I want all the solved tasks given to me”

I was interested in observing the mathematical classroom communication in each case. How did the teacher ask questions, listen and respond to the pupils’ questions? How did they deal with the ‘triadic dialogue’ (Lemke, 1990, here: Wells, 1999:167), which is the prototypical form of discourse consisting of three steps: an initiation, formed as a question, a response, where a pupil answers, and a follow-up where the teacher gives feedback to the response. The three teachers I followed all wanted to improve the pupils’ understanding through open problem-solving, but I wonder whether the teacher and the pupils looked for the same understanding; whether they found the same meaning in solving the tasks. In all the classes, both the pupils and the teachers were unfamiliar with open problem-solving, both in terms of methods, content and the actual tasks. Tina and John told the pupils that it was for practising for the exam situation, while Adrian did not provide any explanation. In all three classes it was common for the pupils to work in groups. During the group work, I followed the teacher, but I often stayed a little longer with the camera in the group after the teacher left, to listen for reactions and to see how the pupils continued and how well they understood what the teacher had explained or asked them.

The guide for the interview, made just after the lesson, was different for each teacher. I wanted to know if any of their beliefs had changed or how they practiced the input from the course. To ask for that, I used some of their own comments from the first interview made before C-02. Certainly, there were uniform questions about how they felt about the recent lesson, and how it was related to C-02. They all responded that the course had inspired them and mentioned some factual elements they used in their classes afterwards or for the pupils' parents, when they were invited to the school, e.g how to describe things, their own tasks made with colleagues, and how to deal with open problems in general. I noticed agreement about problems of non-existing cooperation with colleagues. They all referred to difficulties with communicating their own excitement when they returned from C-02 to their colleagues at the school, and expressed a desire to have colleagues to collaborate with. This result is mentioned in other research as well (see Krainer (Krainer, 2001), Clarke and Clarke (Clarke and Clarke, 2005)); this is not a topic in this study, thought because I was interested in the teachers' mathematical communicative practice, in how they transposed the ideas from C-02 to their own practice and in how they reflected upon them.

5.3 Case study 2

I followed Adrian in case study 2.

Adrian wrote me a letter before my visit, in which he asked for my help. He wrote about his uncertainty:

> *The pupils still have great difficulties with open tasks. I do not doubt that a change will happen and they are getting better, but they are - and I am too - very uncertain. Why? Well, they panic if there isn't any example they can reproduce. I get nervous and uncertain about how much the pupils learn – what happens with more open tasks or more closed tasks. Please, help me! (my translation)*

Of course, this letter influenced my observation; now I knew how uncertain he felt.

Adrian began the lesson by introducing a new topic about equations and math stories. The tables in the classroom were placed in a horseshoe shape, and Adrian walked around in the area between the tables talking with pupils and writing on the blackboard when necessary. The pupils adhered to the social norms of showing of hands and listening to each other. The atmosphere seemed friendly. Adrian started by questioning the class about equations.

[3] ADRIAN: What is an equation?

[4] PUPIL (1): An unknown and an equality-sign

[5] ADRIAN: Can you explain what an unknown and what an equality-sign are?

[6] PUPIL (1): Isn't an equality-sign if something is equal to another thing? Unknowns are x and y

[7] ADRIAN: OK (he writes on the blackboard: '= x and y') equal with and x and y. Then we have an equation?

[8] PUPIL (1): Numbers should be a part of it as well.

[9] ADRIAN: Numbers should be a part. OK, I find a number (writes 4 after the 'y') four.

[10] PUPIL (1): The Square is equal '3x'.

[11] ADRIAN: You want this one (writes: 'y = 3 x' on the black board)

(…) The pupil shook his head and did not say anything; he looked a little confused.

[26] ADRIAN: To be more precise. What do x and y means?

[27] PUPIL: A number.

[28] ADRIAN: Is it just a number? What kind of number? (Points at 'y')

[28] PUPIL (2): Everything.

[30] ADRIAN: It can be everything. Three cows are equal to this (points at '3 x'). (…) If x is a cow, what is y?
[31] PUPIL (3): Then y is three.
[32] ADRIAN: Three what?
[33] PUPIL (3): Three cows.
[34] ADRIAN: Yes, that is right.
[35] ADRIAN: Yes, so it is. It is a mathematical representation of something else; that is your tasks for today. (…) I shall give you an example. If we have (writes '1 x + 1' on the blackboard)… what could be the story for that… I don't know… maybe, in all sports, we have a field with a number of players and one judge. Do you understand? That kind of story, now try to make stories about the formulas I made.

Adrian told me on the way to the classroom that he wanted to connect equations with functions through stories. Adrian made some tasks himself, where he wanted the pupils to make stories from his formulas, as he called the tasks he made (see appendix IV). Adrian's story about the three cows caused the pupils some trouble. In his introduction to the first group work, he was not specific or precise in his explanation of what he wanted the pupils to do, and what kind of stories he wanted them to produce [35]. The pupils were to solve the tasks in groups of four. The pupils had good manners and tried to do what Adrian told them. Four groups were established and started to work, but they were all very confused and did not know how to start or what to do exactly. Therefore, Adrian had to run around and explain to the groups what he meant them to do.

The following episode was part of the communication in a group of four girls (whom I shall call Anna, Betty, Cecilia and Doris) as they discussed the solutions to the tasks: "Equation: Tell a story about what the formulas could represent: '2x ='; '37x =' or '43.25x ='". Notice that the expressions were neither equations nor functions. During the episode, the teacher visited the group every now and then. The whole episode took about ten minutes.

[41] CECILIA: We are going to tell a story
[42] ANNA: It could be a multiplication table for the number 2?
[43] CECILIA: It says we should tell a story
[44] BETTY: Well, two cows on the field...
[45] ANNA: Yes, two cows… two of something… that was what he said, I think…
[46] CECILIA: Is it how we should do it?
[47] BETTY: Yes, I think so
[48] ANNA: But the multiplication table for the number 2…
[49] DORIS: But 2, 2 times x…
[50] BETTY: I think it was what he said, two cows
[51] ANNA: What about asking him?

ANNA called Adrian; everybody waited in silence for the teacher to come.

[52] BETTY: Do you want us to write the multiplication task for the number 2 or two cows? Several of them spoke at the same time
[53] ADRIAN: Yes it is correct, it is correct, I will say…
Several of them spoke at the same time.
[54] ADRIAN: Yes, I will say it can be used when you make an expression of the multiplication table for the number 2, that this expression...
[55] ANNA: Is that what we should do, expressions?

[56] ADRIAN: Yes, you should find an expression
[57] BETTY: But you said we should make a story?
[58] ADRIAN: Yes, and a story.
[59] ANNA: This (?) multiplication table for the number 2
[60] ADRIAN: It is a kind of story too, but…
[61] CECILIA: I don't understand (?) a good story?
[62] ADRIAN: Yes
[63] BETTY: What if there are two cows?
[64] ADRIAN: You could make a story that says: When I was a little boy in first grade, this table was one of the first tables I learned. (Mumble) No, I don't know… what are you thinking of?
[65] BETTY: Is it an expression we should make… no, I cannot (bangs the table with a gesture of despair)
[66] ADRIAN: You could say that I evaluated it as a good story, if you say it is the multiplication table for the number 2, and then I know what you mean, but others can come up with alternative stories.
[67] ANNA: But isn't there (?) I still don't understand it…
[68] ADRIAN: What are you thinking of?
[69] ANNA: I don't know
[70] BETTY: I think of…
[71] ANNA: What about this…
[72] ADRIAN: You are allowed to make different stories, it isn't that. You should just… we will talk about it in the plenary session, if they are acceptable…eehh…
[73] ANNA: Is this how it should be? What can it be?
[74] ADRIAN: Yes, what could it be?
[75] ANNA: Or should it be a story too?
[76] ADRIAN: Yes, an expression for…
[77] BETTY: You should write expressions instead of stories.
[78] ADRIAN: Yes, it's right. But I want stories, but you are right. I will remember for the next time… an expression for…

Several of them spoke at the same time and Adrian left. The girls wrote "the multiplication table for the number 2", and after a short discussion about a story for '37x =', they wrote: the multiplication table for the number 37.

This short episode showed some of Adrian's troubles. Adrian's first task was of the form '2 x = ', which seemed to be a mix of an equation and a function or none of it. He wanted the pupils to make stories about the representation. In my interpretation, his tasks and wishes for the teaching did not fit well: He wanted the pupils to make stories that show how equations can represent contextual situations, and in that sense build a bridge to functions, but his tasks did neither contain equations nor functions and it was not clear at all what he meant by a 'mathematics story' [64]. His introduction was not clear to the pupils, and a question is whether it was clear to him or if he had the necessary mathematical skills himself as the difference between x as a place-holder or x as a variable. To say that x can be replaced with a cow in an equation was surprising for me and confusing for the pupils and bore witness to a lack of confidence; did he have enough mathematical knowledge to know what he was doing?

It was difficult to understand what he wanted the pupils to do, which Betty told him quite a few times [57] [65] [77], and it seemed hard to know what kind of stories he expected. Adrian said [66] 'You could say that I evaluated it as a good story', which indicated that he accepted the result. The problem could be that he was unaware of his own communication as a mathematics teacher and what he wanted the pupils to do in a mathematical way [78]. He made the tasks when he prepared this lesson, and his imagination of the results was maybe clear to him at that stage, but it seemed that the tasks and the examples did not help the pupils to 'go beyond the equation' as he told me that he wanted before the lesson.

The next task '43.25x =' created new troubles, as we see in following communication:

[116] BETTY: Now we have used the multiplication for the number 2 and 37, why can't we use the multiplication for the number 43.25?

[117] DORIS: Yes, I see what you mean.

[118] CECILIE: How can we make a story out of this. Then we have to…

A ten seconds long silence, they just looked at each other and a little confusion seems to spread.

[119] ANNA: It will be difficult for us….

[120] CECILIE: Oh yeah…

Another period of silence for 5-10 seconds.

[121] DORIS: When will you use this?

[122] BETTY: I will use it in the multiplication table of the number 43.25.

[123] DORIS: When did you ever use that?

[124] BETTY: Never.

[125] ANNA: What about some shopping? When will it be exactly 43.25?

[126] DORIS: Don't you think it is more usual in shopping than in the multiplication of the number 43.25.

[127] BETTY: Yes, I think so, but…

[128] DORIS: The probability is bigger in shopping than…

[129] CECILIE: If we say we start with one. If we say we buy (?) and what is the price of that? We play; let us say a bag of coffee. Let us say it costs 43.25.

[130] ANNA: No, you have to buy two things because when you go out shopping… (?) you always buy more things.

[131] DORIS: But I only think it is a bag of coffee, which costs…

[132] BETTY: I don't think we should use energy to discuss what the actual price is.

[133] ANNA: But one litre of milk costs…

[134] DORIS: If we say that a bag of coffee costs 43.25 then we should multiply with…

[135] ANNA: No, you bought a… what did you buy more? Chocolate for dkr.20… no, no, I don't have more energy for this. Come along with a new suggestion.

[136] BETTY: It is used when you buy things.

[137] ANNA: How will you use it when you are out shopping?

The discussion continued without much consensus.

[155] ANNA: When do we need this?

[156] BETTY: Does it mean anything? It is unimportant? It is the point he wants.

[157] ANNA: What is the point with all this?

[158] BETTY: Let us say the multiplication table for the number 43.25.

[159] ANNA (calls the teacher, saying): We are sitting here, and we are very stupid.

Adrian arrived.

[160] BETTY: Could it be the multiplication table of the number 43.25, or is it too unrealistic?

[161] ADRIAN: It is a little unrealistic, I would say, maybe, the multiplication of the number 37 isn't so usual either.
[162] CECILIE: But it has to do with the form.
[163] ANNA: Maybe something with shopping?
[164] ADRIAN: Yes
[165] CECILIE: Look, how strange…
[166] ADRIAN: Yes, you were saying, you were out shopping?
[167] ANNA: You talked about application, we say shopping.
[168] DORIS: I would rather say something with buying coffee.
[169] ANNA: But you don't multiply in the right way…
The discussion went on again.
[191] BETTY: Do you know how long time we spent on this task (points at '43.25x =')
[192] ADRIAN: Did you used enough time on it? Yes, it is so – I find it reasonable with the multiplication table of the number 43.25; I don't know how many, who is able to remember it … but it is a result.
[193] ANNA: I don't understand anything at all.
Adrian left and the group decided to write the multiplication table of the number 43.25.

Here we have a group of pupils, who knew about equations and could operate with them – Adrian told me before the lesson that they were able to calculate equations because they had had a textbook they used to train this topic earlier – but that day they were given another kind of questions. They gave up finding any meaning [156], they only worked to find results the teacher would accept. He was the authority. Some pupils in the group seemed to have an idea of an application which could give some meaning to the task, but the discussion among the pupils and their confusion made them uncertain. Adrian was not able to counter this uncertainty. He also gave up, and accepted the multiplication table [192] as he had to do with any solution. The tasks were so meaningless and Adrian was so unclear that he had to let 'anything go'. Maybe he heard or saw the confusion, but he did not react constructively to it, maybe he did not know what to do about it. He did not give them any questions or suggestions that would bring them further in their attempts to solve the tasks in an applicative way. In [162] Cecilie pointed out that it had to do with the form of what the teacher asked them for, but nobody noticed her. Making 'mathematics stories' is done even in the lower primary school (Steinbring, 2005); Becker and Selter (1996)), but the way it was presented in this classroom removed the possibilities for recognising anything and left the pupils depending only on the teachers acceptance [66] [192]. They could not trust their own intuition, and in that way they experienced a loss of meaning: they expressed loss of both meaning and understanding [65] [157] [193].

In the second session, Adrian's tasks were of an opposite kind, with stories that the pupils should make equations from. The tasks were not aimed at the oral exam as such, but more on practicing communication in groups to solve open problems. The class was not familiar with open questions or problems. The type of tasks I sent them before C-02 was new to the pupils, and Adrian's tasks were as well.

The interview guide made for the second interview with Adrian, which he saw before the interview:

Were you inspired by the course? In what ways? Did you make any changes after the course? What kind of elements from the course did you bring home with you?

Which inspiration is left? What has been difficult/easy? What decisions did you stick to?

The last time: Open tasks end in many different directions. Is it still your experience? Has there been any changes? Did the course give you any useable ideas?

The last time: The pupils often experienced that they had a break when they worked with open tasks. Is it still the same?

Developing techniques to ask and guide when working with open problems:
How do you prepare and develop questioning techniques? How will you prepare a framework to work with open problems? Is everything equally good? How will you value different solutions? What kind of freedom do the pupils have? How will the pupils get mathematical inspiration along the work?

How do you prepare your teaching with open problems? How do you reflect on your teaching?

What kind of materials do you apply in your teaching? How do you differentiate your teaching? What kind of professional ideals do you wish to improve? Are the pupils more used to oral mathematics now? In what ways? How does mathematical communication take place?

The interview was made immediately after the lesson and lasted for nearly an hour.

Speaking	Commentary
L: The pupils were occupied with doing all the right things and they had some interesting comments along with the work (…) Adrian: (…) I know that the tasks are very open and that it isn't a correct mathematical expression '3x =' L: Then, why did you choose it? Adrian: It is between equations and functions and was what I wanted to focus on. (…)	I wanted to know what made him choose tasks that he knew were not mathematically correct. I wanted to know his intentions or ideas with the tasks.
Adrian: I know it is difficult for them, but I feel I need to twist their arms instead of just asking them to solve some equation. They are so focused on giving a right answer.	He defended his choices; nevertheless it was not mathematically correct and therefore extra difficult for the pupils. His answer made me suggest other ways, but he seemed to be convinced of his own practice-theory.
L: You could ask them to make stories from the equations (…) they had great difficulties. Adrian: Yes, I know they should work hard, but they had time to do it (…)	He seemed to be convinced that the pupils needed to get out of the patterns they usually used to solve or conceptualise equations.
L: What did you have in mind when you produced the tasks? Adrian: I don't know if it was thought through. It is the results of playing such a quick-writer and trying to make it more than it is. (…) L: It has some consequences. Adrian: What would you suggest I wrote instead? L: I will think about it and write you (…) as I see it,	Because Adrian wrote me the letter and asked for help, I tried to tell him where he could reflect on his own action: how he made the tasks, what were his conjectures for the lesson, how should these tasks help that goal? He felt stressed and to me it was too difficult with the two agendas. I promised to write him a letter, where I would explain some of the reasons for his stress to him, but the interview was certainly influenced by my double role.

you try to transcend barriers, but you leave the pupils without footing. Adrian: I see two ways of doing it; you could either take them step by step through the problems and make them like mathematics without thinking of any kind of application or you could dare to say, I throw all the balls in the air and then try to catch some of them; and it is my hope that they catch some of them and then say, now I will use more energy – because I think it is about using energy. (…)	He admitted that the tasks was made as some quick-writing, and not thought about much, but he never discussed what such tasks did to the pupils. He defended his tasks without real arguments. His belief seemed to be that ‘reproducing examples and finding one right answer to some tasks’ were not enough. He wanted the pupils to change their beliefs so as to realise that mathematics is more than that, but what this ‘more’ is, he was not able to tell or show. I wonder if he was aware that this way of doing it only changed the pupils’ incentives to find out what he wished. When I asked him about his own ideas for how the pupils should answer his tasks, he was not able to answer.
L: (…) In a way the course has put and extra load on you, which you are unable to administer. Adrian: Yes L: Without giving you enough background. Adrian: They supported my knowledge, but my ambitions are problematic because they are so far away from practice.	He said he wanted to use the course more, but the conditions at the school limited his options, and he felt alone. In that way the course stressed him even more.
L: They support you in continuing doing stressful work? Adrian: I don’t think you could arrange to give me those tools; it is so far away from my daily life. It has to do with the contradiction between my didactical possibilities here and what I believe in. (…) In that way you can say that the course is a thorn in my side, but I don’t see it that way (…) but of course you can say that it is a load to be confirmed in something you are not able to practice.	‘His ambitions are problematic…’ was his own way of expressing it. I told him that I saw how the course did not bring the class any good, but he did not see it in that way.

After my visit I wrote him a letter, in which I explained some of the consequences of his teaching as I saw them, but I was unable to help him any further. I am convinced that he wished to do a good job, but his ideas were so vague that they did not work well in practice. He agreed with my conclusion, that the course only gave him visions without competencies to practice them: “(…) the course is a thorn in my side; (…) it is a load to be confirmed in something you are not able to practice”. His beliefs about the way to achieve the aims were not realistic or ‘thought through’ to use his own words. His two ways of ‘doing it’, I understood as beliefs, but it seemed that he expressed them using buzzwords. He used phrases that expressed a very limited understanding of mathematics and mathematics teaching, and his metaphor with the balls in the air was so broad that it was difficult to understand what he meant.

As for my interpretation of the observation, he needed to learn to understand and reflect on what his practice did to the pupils. He said that he had watched so much good teaching, but I saw that he did not know how to do it himself; at least not in the lesson I observed. He was motivated to change his teaching, but he had no plan for what the changed teaching should be like.

My conclusion of this observation and interviews was that he used buzzwords almost as empty phrases; he knew the words, but did not know how to transpose them into practice, maybe because he had no idea of what they meant. He expressed his ambitions also as empty buzzwords. He could not see the consequences of his own practice, but blamed outside circumstances for his problems. He was the only one in the first interview who mentioned that his pupils were not ready for the exam – he was not talking about himself. As I see it, he tried to camouflage his weaknesses in mathematics and mathematics teaching with all those empty buzzwords, they worked as a kind of smoke screen. He was afraid to use too much energy in his teaching, he said, because it could cause a burnout. The letter he wrote me made me wish to help him, but it was not possible for me to show or tell him how to use his energy differently to create a more effective mathematics teaching. In his letter to me, he wrote that the pupils wanted tasks to reproduce, but what I saw was that they need to understand and see meaning in the tasks he gave them. Because he was unable to give the pupils any example they could use, they were left in a void of uncertainty. It was not because the pupils just wanted to reproduce, but did because they did not know what else to do, or could they see any meaning in the tasks. In the first interview, Adrian said:

> *"The children have their own language and my tasks have mine, which the pupils have to trouble-shoot before they can understand".*

This sentence acquired another meaning to me after observing his teaching. Adrian was undoubtedly a well-liked teacher, and the pupils were doing their best to find out what he meant, and in a way that became the project: Finding out what the teacher meant and satisfying his wishes.

Adrian was satisfied with the course, but what did it give to him? If I had not conducted the observation I could never have guessed that C-02 could influence teaching in the way I observed in Adrian's classroom.

For a summary of Case study 2, I will conclude that the obstacle observed was the use of buzzwords in empty phrases. This means that the teacher knew the 'right didactical' words, but did not understand how to practice the conceptions they represent. In the booklets from the Ministry of Education, a lot of didactical words (buzzwords) are used without any explanation. To fulfil the requirements, teachers often try to find out what they cover, or maybe just use the words without thinking about what they cover at all. One of Adrian's problems was, as I saw it, that he talked in such buzzwords without clear understanding of what they meant. The result was that the pupils in future found it difficult to understand what he meant. Yet, working with open problem-solving became even more confusing and requires maybe an even better understanding of mathematics and mathematics teaching than teaching without open problem-solving. In this case we saw an example of how confusing open problem can be when the teacher's mathematics skills were not sufficiently developed.

5.4 Case study 3

In case study 3 I followed Tina.

Tina's class continued a piece of work that ran for two lessons before I participated. The pupils knew more or less what was expected of them throughout the lesson. Tina did not have to explain much, she just asked the pupils to continue their work. The class had a short lesson compared to the other classes; its length (45 minutes) was normal for the school. Therefore, they needed to have several lessons in succession if their work lasted for more than 45 minutes. The lesson for the observation was number three of four lessons in such a sequence.

The class worked in groups of one to three pupils; groups they chose themselves. The pupils knew what group they belonged in, they had their tasks as hands, and most of them had already started working. Tina had produced three tasks for the sequence; two of them were very similar to the chocolate box task. The first one was about a candy producer and a pyramid box, while the other was about a family who wanted a new garden. The third one was about buying pizzas and of the type where the pupils had a lot of information from which they should formulate questions on their own as well as answer them.


Pyramideæske.

Pyramider har gennem tiden facineret mange mennesker. Pyramideformen bruges i mange sammenhænge.

I skal fremstille en æske, der netop kan indeholde en pose tvist. Æsken kan have form som en pyramide.

En fabrikant fremstiller emballage til forskellige formål. Han kan sælge emballagen for 1,10 kr pr. stk. og tjener derved 20 øre.
Hans gamle maskine kan fremstille 4 stk.emballage pr. minut, men han har på en udstilling set en ny maskine, der kan producere 10 stk. pr. minut.
Maskinen koster desværre 235000 kr, så han ved ikke om det kan betale sig at købe en ny.
Hvad vil du råde ham til?

Fremstil nogle pyramider af papir (se bilag) - eventuelt også andre størrelser.
Find pyramidernes overflade og rumfang.

En fabrikant har til sit produkt valgt at bruge en pyramideæske som indpakning
De skal pakkes i papkasser. Hvilke mål syntes du at papkassen skal have?

Figure19: The Pyramid box

The text translated as 'Pyramid box':
Pyramids have fascinated many people through out the ages. The shape of a pyramid is used in many situations.
You are going to produce a box that can hold a bag of candy. The box must be shaped as a pyramid. The manufacturer produces packaging for different aims. He sells the packaging for DKK 1.10 each and earns 0.20 for each. His old machine produces 4 units of packaging each minute, but at an exhibition he recently saw a new machine, which was able to produce 10 units a minute. Unfortunately the new machine costs DKK 235,000, and therefore he does not know if he can afford to buy it. What would you advice him to do?
Produce several pyramids of paper (look in the appendix, where you will find a template) in different sizes. Determine the surface area and volume of the pyramids.
A manufacturer chose the pyramid for the packaging of his product. The pyramid boxes have to be packed in boxes. What size would you suggest the box should have?
Offer a plan to decorate a pyramid – preferably geometric figures. (my translation)

The class had an extra room next to their classroom which they could use. Nine pupils went there to do their work; the remaining ten pupils stayed in the classroom. Tina took a fast check among all the groups to help and inspire them in their work – or to make sure that everybody knew what to do. One of the groups I followed consisted of three boys (whom I shall call Anton, Bo and Christian). They were less disciplined than other groups; they talked very loudly and had not produced much yet. Tina helped them and suggested very firmly that they should find the area and volume of different things, for instance the pyramid, to know how much candy it could hold, and how much paper it took to make it. They began with the construction from a template.

[75] TINA: Do you remember what you did last time?

Nobody answered this question.

[76] TINA: If one of you produces the pyramid, maybe the others can do something else. What about finding the volume of the pyramid?

[77] BO: For what reason?

[78] TINA: To calculate how much cardboard to the biggest volume. If you then want a bigger volume, how much cardboard will you need? In this way, you can think about different things.

Tina left the group and Anton started to investigate the pyramid, which was made very quickly from the template.

[79] BO: Why don't you just measure it from here to here (point at the height in one of the triangles that is a side in the pyramid)

[80] ANTON: It goes inward, doesn't it?

[81] CHRISTIAN: Now, let us see what we need to take away…

Bo and Christian measured the height in one of the triangles, which was a side in the pyramid.

[82] ANTON: You can't do that (?) are you stupid or what?

[83] CHRISTIAN: We can do it. We struggled with doing so many other things.

[84] BO: It is the same all of it…

[85] ANTON: Look here, first we have this (show the height in the pyramid, which he holds in his hand) and then we have this (unfold the pyramid and point at a height in one of the triangles). You can see that it is not the same height. Now you measured this here (point at the height in the triangle)

[85] BO: OK, that was what we did…

[86] CHRISTIAN: But it says here in the paper how to do it (points at the task). Look here it is straight up in the air.

[87] BO: You measure the height from here and up

[88] ANTON: I hope you see where it is, right? (Points to the top of the pyramid)

[89] BO: Are you sure?

[90] ANTON: Yes, I am, otherwise you measure it higher than it is.

[91] BO: Can I try? (Takes the pyramid and the ruler and tries to measure holding both in his hands)

After some different imprecise measurements and further discussion about the numbers 10 and 12, they agreed about the height to be 10 cm. Tina arrived at their table.

[118] ANTON: I learned that there is a difference between the height you measure when the pyramid is made or unfolded, but the two of them don't understand that you cannot just measure this side (points at the triangle).

[119] TINA: How do you measure it, then?

[120] CHRISTIAN: Yes, show her (push the ruler and the pyramid to Anton)

Anton tried to put his ruler into the middle of the pyramid and looked for the height.

[121] TINA: From where does your ruler starts?

[122] ANTON: From the bottom, it starts with zero.

[123] TINA: Does it starts at the edge of the ruler?

[124] ANTON: No, it doesn't and therefore we did like this (shows with the ruler on the edge of the table)

[125] TINA: Is this a certain way to measure?

[126] ANTON: Maybe we should lend a ruler triangle

[127] TINA: That was maybe an idea (she looks around and finds one on another table, which she gives to Anton)

[128] ANTON: Yes (puts the triangle into the pyramid to measure the height from the table to the top of the pyramid) it is ten.

[129] TINA: OK, with a bit of good will it is ten

[130] BO: Yes, with a bit of good will…

[131] CHRISTIAN: And good wills we have…

[132] ANTON: Maybe it is 10.013…

[133] TINA: OK, we conclude that it is 10. And what should you find more?

Tina left the group and they calculated the volume and area of the pyramid by using formulae written in the task and with the height of 10 cm for the pyramid. The task had some degree of freedom for deciding how much cardboard to what volume. What we followed here was how the pupils determined the height of their pyramid. They needed to know about the height to made further calculations of the volume (1/3 x h x G). They were practical and used only one measurement method. It is unknown whether they knew of a more theoretical method. They were focused on the problem, and discussed what the height in the pyramid actually meant; this was the central topic for the discussion. They were somewhat imprecise when they measured it. They did the measuring in a 'trial and error' manner, and found that it was difficult to be precise in that way. The solution the teacher helped them to find was one way to achieve a more precise measurement, but she did not ask them or tell them how to calculate the height in a theoretical way e.g. using equilateral triangles. It seemed as if she did not listen to Anton when he asked her about the height [118]. She was more focused on activating the group than on listening to their discussion. The group's discussion and measurements seemed to me to be the right basis for a challenge in that direction, to solve the problem through a calculation. They already had experiences with different solutions from the practical measuring, and they needed to know how to calculate the area of the cardboard for other volumes of pyramids. They could have used an equation to compare with the use of cardboard; it would at least have helped them. Instead of talking about the different heights, it became a question of rulers and how to use them. Anton [118] explained the problem, but Tina never reacted to it, the ruler and the measurement won the attention. Furthermore, Tina accepted their results with an 'OK' [129][133]. She asked the questions, showed them how to practice the measurement and accepted the solution.

In the visit, I talked with more of the pupils about their work with open practical problems. They answered very identically that they liked this way, where they could formulate some questions on their own and skip past the difficult things, even though they admitted that it was not always possible. They said that they felt motivated to come up with their own questions; they experienced good challenges, which required some effort. They expressed that this way of working required more effort from them than working with normal tasks from the textbook did. The tasks they used in this class were not of the 'problem posing'-type, but more of the kind of the 'soap' or 'chocolate box' tasks. The pupils had to find a solution to a problem that was given, or one they made themselves. The task with the pizza was similar to the 'weather task'.

The interview guide for the second interview with Tina, which she saw before the interview:

The last time: Difficulties in doing things from other sources than the textbook. Is this still a problem or how do you experience it now? Have there been any changes? Did you get any ideas from the course? Are you able to use them? When the pupils express that they want to 'find out', what are they thinking of? Are they still looking for solutions to satisfy the teacher? What are the skills they want to master? The last time: The classes in lower secondary (and the pupils' parents) had resistance to new skills and want what they could recognise. Are there any changes here? When you work with open problem solving: Is anything changed? How will you rate the different solutions? What role does the teacher have when the pupils work with open problems? How do you prepare the lessons where you work with open problems? What kind of freedom do the pupils have? How will the pupils be mathematically inspired along the work with open problems? Are the pupils more used to speak about mathematics now? In what ways do they speak? How does the mathematical communication take place?

The interview made immediately after the lesson:

Speaking	Commentary
L: Let us begin with the course. What did you take home with you?	I wanted to hear if she felt she had benefited from the course and in what way.
Tina: Yeah, I had a lot of thoughts because of that course, and some good ideas for my teaching too. Still, it is difficult in lower secondary. Working with open problem solving is something we do besides the textbook, which is still our main source, when new skills are taught. Because then I know that I have taught them what I have to, and then we can work with the other things.	Her organisation seems to have changed, but the textbook still played an important role.
L: It means that you are satisfied with having a textbook in lower secondary, when the class is introduced to new topics? Tina: Yes, then I know that they have done some tasks and should know the topic.	
L: When I asked the pupils in your class, they said that they liked the open tasks. (…) When I asked them why, they answered that if they could choose themselves, they felt safer – and more motivated. Do you think you have been successful in this lower secondary class of yours?	It was difficult for me to see, whether it was her or the pupils who wanted the textbook. By success, I meant if the pupils learned some mathematics by working with open problems.
Tina: So far, I think they did benefit from trying to work with them (open problems) in this way, it is certainly the way they should do it in the exam. L: What succeeded for you?	I was interested to hear whether she could explain what they were doing in terms of positive progression.
Tina: The first time, when they solved the tasks from you, they found the tasks difficult, they found that the first one (the closed one) was the easiest and the most fun, and the one they learned the most from. We worked on from that point and talked about what kind of mathematics the open tasks needed and what we learned from that. In that way	The pupils worked with open problems, but when

they learned about open problem-solving. They are more conscious about their way of working, I feel. But they are still not very good at it; they still make their own tasks like the tasks in the textbook.	they had to make questions themselves, they used what they knew from the textbooks.
L: You say that your hypothesis is that they are still rooted in their habits (…) what are you doing to make them less rooted in these habits?	I was not surprised that the pupils followed their habits, and therefore I wanted to know if she did anything to help them further and if so how.
Tina: I am not so good at planning and to say: "Now we do it in a different way"; I still do the teaching in a very traditional way. I guess I don't dare, I haven't been good at it. L: What is the reason for not daring? Tina: I am afraid that they will not learn the syllabus if I don't teach it in a traditional way. I become uncertain. If I teach them, I feel that I did what I could. I know I use to answer the pupils by telling them what to do, instead of asking questions and finding out what their real problems were. (…)	This answer was a reflection upon what was going on. Her own habits, which were necessary for a teacher to look at but difficult to change, she was conscious of. She was aware of that too, that she knew, but did not have enough tools to dare to change them. She described her awareness as a way to reflect, and I wanted to hear how she reflected on her own practice.
L: What do you mean when you say that you 'work with yourself' and your way of asking questions? Has anything happened, now you are more aware of it? Are you aware of what you did before and what you are doing now?	
Tina: I am more like a coach instead of… L: What does a coach do? Tina: Gets them to… they often experience when they need to describe their problems that they are able to solve them. When they express and explain their problems, they become more aware of the problems instead of just getting a method, where I tell them what to do. If I just tell them, they just do it, it is easier, and then I just walk from one pupil to another…, yeah, I don't know. (…) and then I think, when I see a helpless pupil after I taught them from the blackboard, that they didn't learn so much from that either. Maybe it would help her to work in another way; I feel that it is difficult to help them further than they are.	Again she reflected upon her own teaching and beliefs, the changes and the pupils' reactions.
(…) L: Did you like to be at the course? Tina: Yes, it was great, but when I returned, I felt like the worst teacher in the world… because I don't do this or that or that, and I never thought of it because I didn't know about this way of teaching.	She felt she was a 'bad teacher' because she was aware of another practice, but as she said she didn't know how to do it differently.
L: And remember it says 'use it or lose it'. Tina: I know, I need to maintain this, and the first weeks I read a lot in the papers and the articles from the course, to keep me on the track, and made a plan for when we would work with open problems and when with the textbook. But for it to become a natural part of my every day teaching, I need to change something in my mind and think of it as a natural element (…) I was glad that the course had a practical part and that we met later as well. (…)	If she does not practice now and continue what she started, my experiences tell me that she will properly never do, therefore I started to moralise a bit.

L: Did you change your way of preparing your teaching? Tina: Yes, to find tasks that I could use, I needed to… thus, it is hart to find those tasks. L: Yes, the good ones… Tina: Yes, but I learned that even though the task wasn't so good, it could open for a good discussion. As you say, if there is a relevant context, then it is better. (…) L: How do you imagine continuing the development? Tina: I need to practice more and hope I will improve.	If the teachers fail to change their way of preparing, very little will happen. The crucial thing was to know how to prepare teaching in the new way; what to do. Therefore I asked this question. A task can be good in practice, even though it is not so good in writing.

When Tina returned from C-02, she decided to organise the lessons so that she practiced what she had learned. She planned to work with open problems twice a week, and she used the textbook the other two lessons. She told me in the interview, how she explained to the parents what she did and how. Furthermore, she said that C-02 gave her many thoughts and ideas, but still she found it very difficult to teach that way in her lower secondary class. She used it in the primary classes, where it seemed easier. The older pupils were more conservative, she said, they did not want to change their habits or rituals; it made them uncertain, she said – and herself as well. What I saw was that the change already was accepted with pleasure by the pupils.

She told me in the interview that her way of communicating was more suggestive than questioning. Her tasks were easier to understand than Adrian's and the environment she created made it possible to ask challenging questions, which could develop and challenge the pupils' mathematical skills. Before C-02, she did not know how to use open problems in her ninth grade, but now she felt that she had some success and noticed that her pupils expressed appreciation. Tina expressed that C-02 had already helped set her on track to work with open problem-solving in mathematics.

Tina was a young teacher, whom I found to be open and with a professional attitude towards dealing with new ideas. She was focused on the things she did not change, and it seemed that she felt a little embarrassed about it, because she mentioned it several times. I repeatedly asked her what the pupils did differently and it made her explain that the pupils were in fact practicing mathematical communicative skills. They still used their own language, she said, but it was about mathematics. Again she was aware of where they need to make progress.

As Tina said in the interview

> *"…when I see a helpless pupil after I taught them from the blackboard, that they didn't learn so much of that either. Maybe it would help her to work in another way; I feel that it is difficult to help them further than they are."*

I am sure Tina shares this conflict, how to teach all the pupils so as to optimal progress, with many other teachers. She wants to be more like a coach and ask powerful questions in stead of just telling the pupils what to do, but her teaching shows that she found it difficult. Tina reflected on her own way of teaching and was not afraid to admit where she saw her own weaknesses and difficulties. She said that for open problem-solving to be natural, she would have to change something in her mind. She talked of training through practice, and that she did not know how to work with open-problem solving before the course, but now when she knew more about it, she wanted to find out how to practice in the best way. She was aware of her ‘problem’ of not listening carefully enough, which I saw as an important step forward. ‘Be patient and listen to what the pupils say’, is a problem for many other mathematics teachers as well.

The course seemed to encourage her to reflect upon her beliefs, and she already used it when she made new plans for how to organise the weekly math lessons in her class.

In case study 3, I focused on the issues Tina had difficulties with; listening to what the pupils said and how to answer them or asking new questions. Tina told me in the interview that she ‘uses to answer the pupils by telling them what to do, instead of asking questions and finding out what their problems were’. This reaction has consequences for open problem-solving as demonstrated in the example.

5.5 Case study 4

I observed John in case study 4.

In John’s class, the tables were placed in a ‘horseshoe’ with an extra row of tables in the middle. John introduced the tasks and methods for the day, which were similar to the way the oral examination would take place, he said. In the middle of this introduction, another teacher came into the class to talk with some of the pupils, which she did for five minutes in the middle of the class; she was the class teacher. When she left, John continued, but the pupils were somewhat distracted. John cut the introduction short and asked them to establish groups of three. This organisation caused some cooperation problems, but finally all of them found two other to work with. The tasks that John gave them were made during C-02 and consisted of a lot of information about football. From the information sheet, the pupils should make their own tasks. With the information followed a list of ideas such as: calculate the distance, statistical calculations, scale of the football ground, area and volume of a football, etc. The pupils read the exposition and tried to formulate new tasks. John walked around in the class and helped the pupils as needed.

John had a calm attitude and the pupils did not have great problems to read and understand what they should do. A pupil asked:

But there are no questions, are we just making some on our own?

That was exactly what the purpose was, but the pupils did not receive any constrains or pieces of advice for the formulation of the questions. The tasks concerned the World Championship in football in Japan and Korea from different views and with varied information. Moving around, I noticed how the different groups came up with ideas. Most groups produced graphical representations of various tables; e.g. a circle diagram, a curve or diagrams with blocks. The group I followed and refer to chose to work with something else. They decided to find out the volume of a football goal mouth by drawing one.

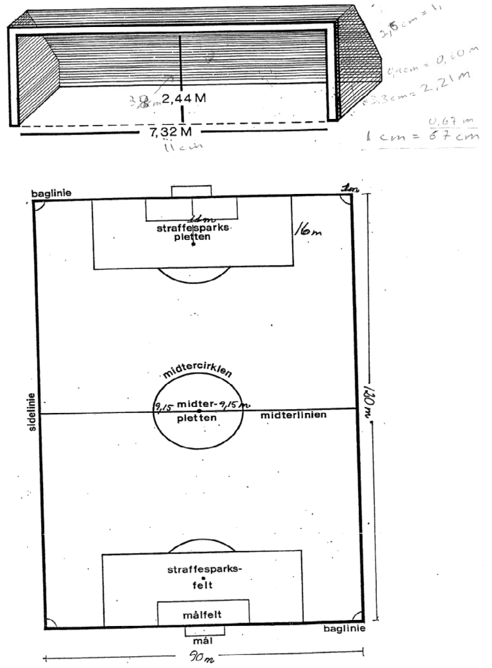


Figure 20: A copy of the football-task

The group consisted of three pupils, one girl and two boys (whom I shall call Line, Carl and Emil). Carl held a football in his lap; it seemed to mean a lot to him. The question they asked was based on their interest in football, so even though they were not very interested in the answer to the question, the process seemed to be more important to them. When I asked them why they chose to work with this task, they answered me:

[146] LINE: Because it is fun

(…)

[147] EMIL: We play a lot of football, me and him, that's why

[148] CARL: Because it is a tough problem and we all know a lot about football

[149] EMIL: And then it is difficult, I don't know how to explain it

[150] CARL: Yes, it is difficult to explain and it is not correct either because we don't have the right measures, but anyhow…

And all three of them continued in an enthusiastic way with the work.

John, the teacher, came to the group:

[163] JOHN: Can you calculate anything from these measurements here?

[164] CARL: We know that this (point at a line on his paper) … and we know that this…

[165] LINE: We found the area and the circumference of this (point at (?))

[166] EMIL: Yes, we found the area and the volume.

[167] CARL: What do you mean by that? (Addressing the teacher)

[168] JOHN: The measures on the drawing, what are they in reality?

[169] CARL: The measures on the drawing?

[170] EMIL: Oh, this one, they mean… (Takes his ruler and measures) mumble some numbers

[171] LINE: Oh, that was what you meant?

[172] EMIL: It is 10-11 cm

[173] LINE: Then we need to find the scale

[174] JOHN: That sounds like a really good idea

[175] LINE: That's the reason why we couldn't … we need to take the measure of these (point at some lines at the paper)

[176] CARL: But we don't know how much these measures are. If we have 32 cm this way then… (…) What about the funny side, we don't get any information about that (point at the sloping back side)

[177] JOHN: What can you do? What can you do?

[178] CARL: We find the scale and then we can… we don't have the length of this side (point at the width of the goal model, which is drawn in perspective) OK; it should be 2.5 cm more?

[179] LINE: No…

Pause, nobody talked.

[180] CARL: No, it has to be a bigger scale… no, this is 11.9 m

[181] LINE: 32 divided by (?)

[182] JOHN: Why did you ask about that?

[183] CARL: What?

[184] JOHN: Why did you ask about that?

[185] EMIL: (measures with a ruler the length of a line)

[186] JOHN: Why did you only go from here to here?

[187] EMIL: Because it was here it stopped

[188] CARL: Yes, it is 11

[189] JOHN: You should keep that thought, Emil, say something

(…)

The problem they worked with was a little strange. They tried to calculate the volume of a football goal mouth drawn in perspective. John helped them to find a scale they could use to find the real measures and how they could find volumes of different prisms, but he never asked them for whom the results were worth knowing; authenticity was missing. They had difficulties finding the right scale because the perspective cheats them [176]. But they accepted some mistakes, something that was too difficult; they knew it but did not bother, because they did not need the result for anything.

John was patient and waited for an answer from the pupils. During the first interview, he was not positive about open problems, but that day he experienced that this working form fits very well into his views about being a good teacher.

His mathematical communication with the pupils was influenced by his beliefs. He did not ask precise questions, but rather commented on some of the things the pupils said or did [177] [184] [189]. This strategy balances on the edge of 'anything goes'. He only asked questions about mathematics or some social reactions; he never asked what the results could be used for. I focus on the situation with the football goal mouth, because I saw three capable pupils who looked for a mathematical challenge. At the same time, their motivation derived from the football theme, and they wanted to combine difficulty with something related to football: "It was difficult and tough". To determine the volume of a football goal mouth did not make much sense to me. The result was not of interest to anybody, and the answer they found was not correct either, but they did not care; and John did not either. Was this a practical mathematical problem? The pupils were preoccupied with doing mathematics, but the result was unimportant. The process gave meaning to them only because it concerned football. And, because they were good at mathematics, they wanted to work 'with something difficult'. But they did not get any tools for finding out how to measure a length drawn in perspective. Their primary motivation was football, not to learn mathematics. The calculation was of course numerical, but the drawing was important and they did not care that they 'cheated' about the perspective measurement. They were not challenged in a way that could be really difficult to them, namely to work with how perspective drawings can in fact be measured, and to explain who could benefit from the knowledge of the results.

John made a short evaluation of the lesson some days later, which he sent me. All the pupils expressed satisfaction and pleasure with the lesson; they felt they learned a lot, they were allowed to make their own choices, they liked to be encouraged to work together and they all expressed that they had fun. Not one of the pupils was dissatisfied. A few of them expressed that they did not like the camera.

The interview guide for the second interview with John, which he saw before the interview:

Were you inspired by the course? In what ways were you inspired? Did you introduce any changes after the course? What elements did you take with you from the course? What kind of inspiration is left? Difficult/Easy? What did you stick to? The last time: Difficulties with teaching in a differentiated way. Is it still a problem? How do you experience that now? Were there any changes? Did you learn new strategies? Did the course give you any ideas? Can you use them? The last time: Uncertainty about whether the pupils learned mathematics. Did you get new experiences? 'Mathematics is unpleasant when it is the same again and again', is it still a problem? Do you work with open problem solving? What are your techniques to guide pupils in open problem-solving? How do you prepare and improve your asking techniques? How do you prepare and set up the framework for working with open problem-solving? How do you prepare a lesson with open problem-solving? How do you reflect on it? What kind of materials do you use in the classroom? How do you take care of the individual pupil? Is everything the same now? How will you evaluate different solutions? What degrees of freedom do the pupils have? How are the pupils inspired to develop their mathematical skills along the course? Development of professional skills. Do the pupils get further experiences in oral mathematics? How does the mathematical communication go on in your class? Are you more confirmed in your choice of tasks for the exam? Is your background knowledge becoming more consolidated?

The interview made immediately after the lesson: John was surprised that he had such a pleasant lesson. Before C-02, he was sceptical about open problems, but admitted that he did not know much about it. Now he experienced that it was pleasant for him, and that it fulfilled some of his wishes about being a teacher. Furthermore, the pupils were active, as he said, most of them with mathematics.

Speaking	Commentary
L: What is left from the course? You had some expectations, which of them were fulfilled? John: Yes, first I will mention some little tasks as 'drawing a bird'. I used it in a meeting with their parents. (…) The period after the course has been interrupted by illness, me and one of the pupils. I	This question would hopefully give an answer that showed how John experienced his benefit. Some specific tasks were used.

haven't done as much as I wanted to do. So I was surprised that they worked so well today. L: And what are you satisfied about? Can you explain it further?	I wanted to hear about his own beliefs.
John: yes, I felt that they were able to find out what I wanted them to do. They read the tasks and chose what to work with. Quite a few of them worked with statistics, I guess it is because we just worked with that topic. The way they started the work, I liked that very much. It was more than I expected and I didn't have to help them much.	John expressed that he was very satisfied with the lesson and the way the pupils worked with the tasks.
L. I noticed that. What is your opinion of the kind of questions they asked?	I was critical of the questions or tasks the pupils made, because they were not authentically related to any the results. Therefore I asked how John evaluated them.
John: I liked them, they were at a good level for them, I think. Perhaps they could go deeper into … they could say, this part we don't need. There was a group that spends a long time just making different diagrams for the same data. It was not necessary… When they work with closed tasks and there are tasks from a to e, they do all of them, b, c and d. (…)	John expressed satisfaction and compared the questions with textbook tasks. I needed to be specific about the results to hear his evaluation of those.
L: I noticed that most of the questions concerned how to calculate. None of the questions were about who could use the results. It was the calculation process, they were occupied by. John: Yes, they can't… when they found the volume of the goal mouth… what should they use that for. It is quite right.	I wanted to know how he prepared (or if he prepared) his questions before the lesson.
L: It could be your next step to work with. How do you prepare such a lesson? Do you consider what kind of role you will take? I am curious about how much you are aware of your own way of communicating.	It turned out that what he prepared he didn't use and what he used was not prepared, but he was satisfied with the results.
John: My role to day was less important than I expected. This was the first time I have tried to give them only lots of data and then ask them to make their own questions. I expected to be more of a starter than I was. I was more like a consultant, which I like to be, and they calculated a lot.	
L: (…) What did you prepare? John: I have to break down and tell you that I didn't prepare very much, but I thought about how to get them started. I thought of some questions to ask them in the beginning, but something else happened. L: I am curious to hear if you would have asked them some practical problems like, how long a distance does a football player run, or…	I asked again about the meaning of the results, at that time I didn't see that his questions to the pupils were only related to mathematics and not to the application. Therefore I became stubborn. His answer just made me continue.
John: Oh yes, I could have made some problems and asked them what would happen if you should imagine to build such a stadium, how much area would it take… but it is more closed as I see it, but still a practical problem.	I was curious about how he developed 'on the fly'.

L: Yes, but to be a practical problem you could say that you need to know what to use the result for. (…) L: Do you elaborate your questions when you walk around, can you tell me what happen when you walk?	In the first interview, John was uncertain about how to deal with open problems.
John: When I walk around I try… I try very much to listen to what the pupils say and try to empathise with them, why do they think as they do? And something changes all the time when I go from group to group. I try to find out how they can make progress, but I don't know… L: Do you find it difficult to change when you try to go into each group? John: No, because I already thought about what I wanted to ask them, and I still try to be flexible in my way of thinking, so I am able to empathise and still keep the overview. L: I noticed that you were listening before you asked questions.	John handed over all the initiative to formulate the questions and tasks to the pupils. He did not inspire them to ask authentic questions. Therefore I got the feeling of 'anything goes' and that was the reason for that remark.
John: I don't want to be the driving force and tell them that they should find the area of a prism then do that and that. It is not something I think about, but it is how I like it in the teaching situation.	He still expressed that he has that problem introducing new topics. I was curious to hear about his practice-theory.
(…) L: The last time, you talked about being uncertain of when the pupils learned mathematics. How are you doing with this issue? John: I don't think so much about it. But I still find it difficult to present a topic at the blackboard. They stop listening. L: Do you have any idea why it is like that? John: I think it is because they have so much project work in the other subjects, and it is not quite mathematics and I find it difficult to combine. I am sure it could be done, but I didn't succeed it this year.	I noticed that he mixed up topic work and project work and tried to ask more specifically about that.
L: Is it something you want to work with? John: Yes, when we work with some topics, let us say statistics, and then develop some materials for how to use them. If we want to use a circle diagram, how do we do that? And then do some project work, where we say that we have a topic… oh no, it is topic work, what I mean is that we take a week with statistic and then see where we can use it in our daily life. L: As I see it, the questions are different when you work with projects or topics. Do you want to answer a question or do want to know some data about a topic. And the requirement for the oral exam is that it should be a practical problem, and then it is more like project work than topics. I am curious to know	Because there was no time for the pupils to share experiences during the lessons I observed, I wanted to know if there would be any.

what kind of solutions you value more than other and how you will tell the pupils. How will you give a final evaluation of the lesson? Have you thought about this? John: No, not particularly, certainly I need to talk with them and then I will ask them some questions, such as where we use all the answers they found. I will tell them that it is totally insignificant to find out the volume of a football goal mouth. (…) John: I don't know if I got more action competence from the course, but I know I was inspired. (…) I think I need experience, which only comes from doing it a lot of times.	

John was satisfied with his pupils and surprised in a positive way. During the interview, he became aware that most of pupils' questions were not useful for anybody, but he did not seem to worry about it. He blamed the textbook they normally used. The 'problem' was that the pupils did not try to solve any tasks where the results could be used for anything; they were not challenged to do so. When I asked him, he said that he realised that the results did not give any meaning; I did not know if it was to please me or if he realised it as well.

His intuition about the cause of difficulties was interesting. Project work could be combined with mathematics through open practical problems, and the way to ask questions when working with project work could be a help to mathematics as well. But he did not see the relationship among them, or as he said, he did not have the energy so far to find out how it could be done.

The cooperation did not work out as being just easy for the pupils, and some of their troubles stemmed from the way John organised the groups. He said that he was aware of the pupils who had difficulties in one way or another. The comments about the social skills came after we discussed a group with two girls and a boy. The two girls talked only with each other even though the boy seemed to have more mathematical skills to offer. The result was that the boy only observed what they did, but when finally asked he turned out to know how to solve the actual problem.

John expressed awareness of giving the pupils time to think, and that was what I saw. The interesting question was what kind of filter he used when he listened. What was significant to him in the mathematical communication? His questions were about the pupils' mathematical strategies or some hints about social behaviour, not about what kind of questions they wanted to solve, or anything about the results. He seemed to want earnestly to find out how to improve the students work and skills. He summarised and repeated that he felt inspired by the course, but wanted at the same time to express that he was still uncertain.

At the time of the interview, he did not know how positive his pupils were about the lesson; whether it had any influence on his decision for further teaching is unknown. He had a patient way of listening, which gave the pupils time to answer, but his attention was mostly on their mathematical strategies. He was not aware of the different mathematical understanding. He could easily fall into the 'anything goes' trap if only the pupils used some mathematical calculations.

The 'obstacles' highlighted in case study 4 was the teacher's awareness of what kind of mathematics the communication and questions concerns when pupils work with open practical problem-solving. I realised that John's questions predominantly were about pure mathematical understanding rather than practical or applied understanding with authenticity in the questions. The openness was only related to asking questions and not to any different kind of investigation.

5.6 Summary of the four cases

From my observations, I conclude that all of the four teachers at least to some extent adopted open problem-solving, and all of them tried it out in their classes with different rates of success. In case one, Mona had positive experiences with using problems with an open character; in case two, Adrian made ‘mathematics stories’, which could be seen as a kind of open problems; in case three, Tina trained the students for the exam form with open problem-solving in groups; and in case four John worked with open problem-solving as well. My observation was focused on problems the teachers had when working with open problem-solving in their mathematics teaching, and how they communicated with their pupils in that situation.

The primary causes for problems included:

- Confusing open problems and questions with non-mathematical problems
- Speaking in buzzwords
- Not listening to the pupils’ mathematical communication before suggesting solutions
- No attention paid to the authenticity of the practical problems

These four obstacles are all associated with the individual teacher, and are as such not generalised. The question is whether they are of general interest at all and whether other mathematics teachers could have the same types of problems. Another question is which of the problems it is possible to meet in the kind of in-service course investigated in this study.

6. Discussion of the pre-study

The question investigated in the pre-study was:

To what extent and in what way did some of the participating teachers adopt teaching with open practical problems in their classes, and what kind of problems did they have, compared with how those problems were addressed in C-02?

The following is a discussion of my findings compared with what other researchers have written about the same areas.

Karpatschof (Karpatschof, 2006) distinguishes between the terms 'empirical object' and 'epistemic object'; the empirical objects are the persons (here: the participating teachers) and the epistemic objects are the object investigated (in this case any professional benefit such as a new awareness or a change in teaching methods).

The phenomenon investigated in the pre-study was to what extent the teachers were able to teach mathematics through open practical problems and what kind of problems they ran into when they did it. The data were the classroom observations combined with interviews, while the conclusions were interpretation of the collected data. Some of this data was empirical and can be described, while the epistemic data was hermeneutic interpretation based on observation. The essential source of information in this pre-study, mostly epistemic data was in terms of the different problems the teachers had.

6.1 The particular in-service course

The catalogue text inspired in the first place the teachers to want to enrol, and at the same time, it should be a text that gave them permission from the headmaster of the school to enrol on the course. When teachers apply for permission to take part in an in-service course, they argue why they want the course and when doing that, they simultaneously create expectations to how the course could inspire them. These expectations are closely related to their daily work. It has been shown that people evaluate an event as most successful if their expectations are fulfilled (Hansen, 2001, p. 40). The catalogue text is formulated by the teacher educator who runs the course, but only in brief terms describing the goals and not the process that leads to the goals. Already at this point, misleading or unrealistic expectations can be created. For example, most of the participants felt pressured to hold an oral exam. This pressure motivated the teachers to work with communication in mathematics; the result is showed in an investigation of the connection between this exam and the daily teaching (Danmarks Evalueringsinstitut, 2002). This is not surprising; we know that exam regulations influence the daily teaching (Niss, 1993). The purpose of this oral exam is to strengthen 'communication and problem-solving' in the daily teaching, which is also one of the four central issues in the Danish syllabus. The teachers were usually not aware of that before I told them on the course. The executive order about the oral exam describes the framework for the exam (p. 3), but the reason for this kind of group exam is not explained in any information from the ministry. In an article in a periodical, we find the following quotation of the chairman for the working group for the guidance of the Educational Initiative, H. Nygård Jensen(Jensen, 1996):

> *It is noteworthy that the introduction of the new oral examination in mathematics is in concordance with the effort to strengthen the communicative perspective of the aim. We hope that this new examination form will influence the day-to-day teaching in mathematics. (1996, p.9, (my translation))*

I conclude that this hope builds on the belief that what is to be assessed will influence the day-to-day teaching. Yet, this 'hidden assumption' was not at all clear to the teachers. Many came to the course only to learn how to fulfil the exam criteria. The teachers were not familiar with the kind of tasks they needed for the oral examination or with how to design and formulate them. It was my impression that the teachers had a clear expectation about what he or she wanted from the course. What they were not aware of was the processes for how to acquire these competencies; they were not prepared for their needs to learn unexpected skills. I saw this pattern as one of the challenges both for them and for me.

What I did on C-02 - I see this now - was to set up activities. These activities were not always connected to a certain theory; instead they were often practiced on what Handal & Lauvås (2002) call practice-level 1. My idea was nevertheless that these activities would prepare the 'teachers' for their daily teaching and their pupils' exam, because they were combined with discussions about constructing educational programs, which could be viewed as actions on level K2 (Dale, 1989) because elements from the Act were included. In the teachers' explicit expectations to C-02, quite a few teachers expressed that they wanted to be able to produce tasks and carry out the exam in accordance with the ministerial requirements. This I see as a three-tiered desire: Curricular knowledge, mathematical content knowledge and pedagogical content knowledge, where pedagogical content knowledge is defined as what

"…represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction". (Shulman, 1987)

Pedagogical content knowledge shares similarities with constructing educational theories, which is what takes place on level K2-3 (Dale, 1989). The problem is that it takes both time and effort to transform this knowledge into competencies. Furthermore, the expectations often cause a conflict between what the teachers want to get out of a course and what they are willing – or able – to do to achieve their goal. We know that learning is connected with 'everything else than customer-friendliness' (Dahler-Larsen, 1997) if the teachers are expected to learn more or other things than they expect themselves. If we want to generate alternatives for the teachers' normal teaching, it requires hard work. Besides we know that learning calls for motivation and engagement, which often comes from feeling safe in a stimulating environment. But the success criterion for the particular in-service course is that the teachers are equipped to teach mathematics through open problem-solving, and not only that they have a good time on the course (short-term goals). The long-term goals are that the 'teachers' are able to develop tools into competencies of action and reflection for their own day-to-day teaching and in that way generate their professional skills; how this will be done is unpredictable and individual for the teacher. In retrospect I see how I tried to satisfy the teachers' short-term expectations more than the long-term goals on C-02. Normally, teacher educators wish that both short- and long-term goals would be fulfilled at the same time, but it is not always possible. It could be that the course description gives higher or different expectations than it is possible to meet. It is difficult to give honest declaration of contents, when the market conditions are breathing down one's neck. But who is to blame when the goals are not achieved? When I read the evaluation of C-02, it could only inspire me to continue and 'don't fix anything that wasn't broken'.

In a Danish research report about in-service education (Grønsved W. et al., 1998), we hear some teachers' voices. The research contains an evaluation of in-service education at the Royal Danish School of Educational Studies. The survey is a quantitative investigation of the expectations, experiences and development of the teachers at in-service education in general. Some of the results from the survey (N = 1186) tell us that 89 % of the enrolled teachers at in-service courses found the course they participated relevant for their pedagogical work in the school; still, there were 11% who did not, and some 30% of the teachers answered that some of their courses were not usable. They gave four main causes as explanation (p.33):

1. In-service courses those are too theoretical
2. In-service courses that do not live up to the expectations given in the course description
3. Not enough new knowledge
4. Disinterested teacher-educator (my translation)

The first cause: Further explanations put the finger on the gap between theory and practice, because the teacher-educator was unable to relate or combine the theory to any relevant practices. The second cause: The content at the in-service course differed too much from the course text in the catalogue and it was too difficult to discuss the content with the teacher-educator as well. As for the third cause, that they get not enough new knowledge: From my own experiences, I noticed often that teachers expressed that they did not learn anything new, they say that they knew it all and that their practice shows it; it could be right, but it could also be that they were unable to recognise the new material. The fourth cause: some teachers claimed that the teacher-educator was too boring and did not provide inspiration for the work that should be done; furthermore they complained that they felt they wasted time at such courses, that the teacher-educator seemed not to be prepared etc.

Compared with the results from my pilot study, the teachers-students expressed satisfaction in their evaluation, which perhaps could be called a test to show whether they were satisfied. They were satisfied with the activities I planned. They liked to be at the course and felt that they got new inspiration. They did not like the IT-part for several reasons that are similar to the results in Grønsved's investigation. It seemed that the amount of theory I presented to them was suitable. But to what extent was C-02 actually theoretical, and how did the course combine practise and theory? The way to implement academic theoretical knowledge in the middle long educations is in no way obvious. Teachers' training is one of the medium-length tertiary educations. Yet, the theory is often presented without any obvious link to the profession or the practical issues, which means that the students fail to acquire necessary theoretical knowledge to develop their teaching competencies (Ministeriet for Videnskab, 2006). On C-02, they were not required to read any theory at all; I mentioned a bit of the theoretical background, but that was all. The teachers did not ask for more, and I did not connect more theory to the activities (theory means here reasearch articles or books). I was pragmatic in my expectations in that I did not expect them to read or relate to any theoretical ideas or frameworks. I was a filter that transformed theory into practice without in fact presenting any theoretical sources for their own further development.

Cooney and Krainer (Cooney and Krainer, 1996) describe some teachers' frustration about not having enough good activities to use with the kids on an in-service course. They had a teacher, Maria, express her priorities, if she were to design the program at an in-service course (p. 1165):

- How can we enable students to see the connections between mathematics thinkers and the problem solvers?
- What kinds of activities can help students see the connections between mathematics and real life?
- How can I create better activities for my students without driving myself crazy trying to find resource materials?

We have here an outline of priorities that tend to be activity-oriented. Influenced by constructivism, teacher education programs have become more process-orientated, meaning that teachers are encouraged to act as reflective individuals (ibid). Later in the same article, Maria gets new insight, as demonstrated by more reflective perspectives and wishes, which showed something about the process she had gone through. The constructivist influence was the researchers' knowledge, which they wanted to introduce to teaching; it was not called for by teachers, and it was difficult to transpose the philosophical ideas into practice (Simon, 2000). Krainer (1994, p. 221, from (Cooney and Krainer, 1996) p. 1171) illustrated through their examples the difference between requirements produced by 'act or perish' and indicate that the goals look differently from different points of view. Simon (Simon, 2000) expressed this clash as a problem:

> *Constructivism offers a fundamentally different way to think about mathematical knowledge and its development. By redefining the human learner and furnishing a framework for understanding the complex process of learning, it has caused a re-examination of mathematics teaching. However, whereas constructivism provides a foundation for reconceptualising mathematics teaching, it does not provide a particular model of mathematics teaching. (p. 217)*

There were some similarities in between 'my' teachers' desires to have ideas that could be used in the classroom and their responses in the interviews I made with them. Like Maria, they wanted to acquire competencies that enable them to cope with the feeling of lacking skills. But they did not know what it required to acquire these competencies without enroling in the course.

When I communicated with the 'teachers' on C-02 I was not asking the powerful questions; nor did I teach how to ask them. I presented tasks for the 'teachers' and asked them to discuss what good questions or problems should contain. The groups came up with lists, which I compiled, but I never discussed with them what the end product would look like or how 'The Good Question' could be formulated. I gave them open tasks to solve and discuss, but we never discussed in plenum how we shared values, argumentation or disagreements. Even the role-playing was left to the group to reflect on and discuss on their own, I only saw the results, which I did not comment on. The comment "How should I know?" in one of the interviews told me how important it was to demonstrate how the theoretical ideas could be transposed into practice and give feedback to solved tasks.

It is not always possible to live out our beliefs, and this leads to a discrepancy between what we want and what we do. This discrepancy can be very difficult to reflect upon for teachers at all levels, teacher educators as well. We all have a filter we listen through; it is our beliefs and values (Thyssen, 1997). The questions a teacher asks are a consequence or practice of his listening and understanding. Furthermore, the questions the teacher asks shape the way the pupils work; it seems difficult, however, to ask the right questions in the right situation.

Black & Wiliam (Black and Wiliam, 1998) investigated mathematical classroom discussions, and their results showed what often happens in mathematics classrooms: the teacher answers her or his own questions after only two or three seconds; silence is not tolerated. Additionally,

> *There are clearly-recorded examples of such discussions where teachers have, quite unconsciously, responded in ways that would inhibit the further learning of the pupil. (p. 11).*

These examples show that the teacher does not want to deal with the unexpected; therefore he or she tries to direct the pupil towards the expected answer. The consequences are that pupils do not even try to think out a response – if they know that the answer, or another question, will come along in a few seconds, there is no point in trying. Open problem-solving and powerful questions that encourage the pupils to develop the mathematical skills fit well together, if the teacher has the necessary skills to reflect her own communication and have tools to practice with. Vygotsky's theory about ZPD requires that the teacher is able to ask questions that help the pupil to develop. The unique, as I see it, is that the good question creates the motivation and energy for further investigation. The question, if asked in the right way and at the right time, creates a zone of proximal development. How the pupil should develop in the zone is not pre-decided and in that way the zone does not exist in itself; it is created by the environment, in this example good questions. The results of the process can be assessed and the developed skills do in that understanding exist, but the process is created in the environment. The power of the learning potential depends on the discourse. Awareness of this kind of discourse is not something that comes only through practice; it comes through instruction with a focus on developing communication through reflection on powerful questions.

I reflected both in and on my own practice, but I did not involve the teachers in this process, and I left it to them to decide how to reflect their own practice and experiences. I gave them tasks to perform, but I never discussed with them what kinds of norms were represented in their replies. In that way, I never discussed with them our different relations to the topics 'reflection on mathematics communication' and 'how to ask powerful mathematics questions'. I failed to teach them well enough by not telling them how it could be done or where they could find more inspiration in the literature; maybe I did not think that part of the workshop through, but at the time I did not see it as a problem and neither did the teachers. I tried to make them reflect on the teachers' logs by giving them a tricky task where the teachers should watch their own questions, but this task was never successful. I was unable to make my intention clear and meaningful.

We worked very much with the concept of open problems and different ways a task could be open, but I was not explicit about the different ways the teacher could encourage the pupils or the different ways to impart mathematics as pure or applicative/practice or as authentic tasks. How to ask the right question to the right person so that everybody feels they got an optimal learning experience is the challenge teachers have to deal with every day. Furthermore, I was not aware of the consequences of my implicit assumptions. The evaluations that the teachers wrote at the end of C-02 were fairly standard for this type of in-service course. In this study, however, I made a closer inspection of the influence from the course half a year after the course. My own implicit assumptions when I 'evaluated' the teachers' teaching became apparent to me, and I realised that their teaching in some ways reflected my teaching on the in-service course. They learned that they should use open tasks, but the difficulties inherent in mathematical communication and in how to reflect on this communication and actions was left to them. I left the reflecting to the groups with the assumption that they could manage, or maybe because I did not know how to teach them this reflection. And what I did not know, I just left to the teachers, who again left it to the pupils. This pattern, to leave to the 'pupils', what the 'teacher' does not know how to deal with, was evident at several levels. The classroom observations were meant as a means to an evaluation of C-02, and I came to see the course and my own teaching with new insight that allowed me to see the teachers' difficulties not as 'mistakes', but more as consequences of what they had learned so far. In the classrooms, I saw very good relationships between teachers and pupils, but where they had the difficulties in teaching mathematics through open practical problems was what I wanted to highlight. I could have chosen to look at successes, which was possible as well, but that focus would not give clues for how to redesign the in-service course.

7. Summary and conclusion

The question investigated in this chapter was to what extent and in what ways the participating teachers adopted teaching with open problem in their own classes.

The ‘teachers’ in C-02 expressed satisfaction with the course and as the teacher educator, I was satisfied as well. Therefore I expected to see some of my views and methods shine through, when I visited the classrooms. But I was surprised when I saw the teachers’ problems when teaching mathematics trough open problem solving.

The problems can be classified as:

- Confusing open problems and questions with non-mathematics problems.
- Speaking in buzzwords.
- Failing to listen to the pupils before suggesting solutions.
- No attention to the authenticity of practical problems.

On reflection it became obvious that one reason why the teachers were satisfied with C-02 was that they had their expectations fulfilled and had tried a lot of different activities, while in fact they did not learn any new tools for solving their real problems.

The conclusion of the pilot study was that the teachers felt that they learned to teach with open practical problems in their classroom, but their real difficulties when teaching mathematics through open problem solving was not a part of C-02; they did not know this and neither did I. My expectation was that when they practiced what they learned in the practical period between the two rounds of the course, they would face some of their problems and we could discuss these problems during the last two days, but this was not the case. Faced with these results, I realised it would be necessary to redesign the course to meet what I saw as the real needs of mathematics teachers who use open practical problems in their teaching.

Confusing open problems with non-mathematical problems and no attention paid to the authenticity of practical problems both have to do with the question whether something is practical in a mathematical sense. In the first case the questions are not mathematical, and in the other case they are only mathematical. In both cases, the teachers call the questions open, mathematical and practical questions. What is meant by this is not clear at all, something I found in the analysis of C-02 that I did not help the teachers understand.

I would say that it is a normal problem that teachers are confused about how to ask mathematical questions powerful enough to be both open and practical, particularly when they fail to understand such questions at all or to know how to ask them. Working with practical problems in mathematics produces its own set of problems and difficulties, because the way the questions are formulated, i.e. the discourse, is so important for how the pupils understand the work with these kinds of problems. By way of example I will mention that the Danish written problem-solving tasks for the final exam include questions about which it is difficult to decide whether they are mathematical or not. Danish tradition is to mix open questions in general with open questions in mathematics. These tasks are formulated along the following lines: ‘Describe what your last two calculations reveal about the area, which the TV program covers’ (National final written test, 2005). Such a formulation can easily be interpreted by the pupils as a request for a narrative about the context, which in this case is local TV – and maybe confuse their teacher as well.

Using buzzwords is unfortunately a habit, which can be observed among teachers as among other people. Buzzwords disrupt the children’s learning quite a bit. If the teacher does not know what the words actually mean, it is difficult for the pupils to find meaning without guessing. In the case where I observed Adrian using buzzwords, I saw that the consequence was that he had to accept everything that came from the pupils such as all the funny multiplication tables. He could not argue against it, because he was so uncertain about what his own words meant. In this way, the consequence of using buzzwords in teaching mathematics can be considered as a general problem.

Not to listen to what the pupils really say is not new either. Black & Wiliam (1998) observed how mathematics teachers answered their own questions after just a few seconds if the pupil they asked did not react in the 'right' way. This practice can be interpreted as a communication problem rooted in an uncertainty about mathematics and in the fact that it is easier to talk than to listen. If the teacher only allows the questions she has planned for, she can stay in the 'safe' area. But it could also be a cultural phenomenon for the teacher or school culture that silence is a problem, that the pace should be quick, that pauses are somehow dangerous.

The four cases have in common, as I see it, that the teachers were not aware about their real problems or above how they could reflect on their own way of communicating. Teaching is highly dependent on communication, but I believe that some of the problems could be avoided if the teachers knew how to reflect on their mathematical communication. Yet, to reflect requires tools to reflect with and competencies to choose other means of communication if the reflection calls upon it.

The big question for the redesign was what an in-service course, such as this one, could actually help. This question can be split up into:

- Which of the findings could actually be attributed to the course?
- How could different teaching address the real problems?
- What are the alternatives?

The pre-study provided a peek into three classrooms and an analysis of one classroom session. The peek and the analysis surprised me and gave ample reason to redesign the course.

PART C: Main study

My research question brings together two main themes of my research. One theme is the successive redesign of an in-service course that the teaching and the expected outcome correlate, while the second is the transposition of theoretical concepts from mathematics education research literature into the practice of an in-service course; what I call a ‘meta-didactical transposition’.

The Research Question is:
To what extent and in what way can a meta-didactical transposition be incorporated into the successive stages of a redesign of the course, and how effective is this redesign, measured by the participating teachers' reactions during the courses?

The methodology employed to answer this research question is design research. Design research consists of an iterative process shaping a design, implementing it and analysing how it works and why (not). Based on the analysis of the results from the pre-study, I undertook a redesign, which was tried out on C-04, C-05 and C-06. For each redesign, collected data were analysed and the results used to change and reorganise the design. I use the term ‘artefact’ for subsequent courses, which is described in the figure of the ‘Osmotic Mode’ (p. 36 and 38). For each cycle a new ‘problem’ in this ‘artefact’ is detected and analysed, a new ‘hypothesis’ for how it can be improved is tried out, and ‘data’ combined with relevant ‘theoretical ideas’ form the basis for a new design, which is applied again, analysed, and so on.

The redesigned artefact/C-04 was a framework based on some guiding principles for the organisation of the course. In this chapter, the redesign will be described together with arguments for the hypotheses related to each of the guiding principles in the redesign, how they were prepared, carried out and at last discussed. Part C consists of five chapters, of which the first three concern the redesigns, and the last two are summaries and conclusions.

The redesign in practice is described through different episodes from the three courses, C-04, ‘05 and ’06. The actual episodes were chosen to demonstrate how the participating teachers reacted to the course design, and they were analysed in relation to the guiding principles and the overall aims for the course.

In the final discussion and conclusion a summarised answer to the research questions is presented.

1. Meta-didactical transposition

The expression 'meta-didactical transposition' is derived from the concept 'didactical transposition', which is first suggested by Chevallard (Chevallard, 1985). Didactical transposition is a theoretical concept in mathematics education, and denotes the process of transferring mathematics as a scientific discipline into an educational subject at school.

Brousseau (Brousseau, 1997) p. 35 describes didactical transposition as a transposition of mathematical knowledge to suit an audience of mathematics pupils:

To teach it, then, a teacher must reorganize knowledge so that it fits this description, this "epistemology". This is the beginning of the process of modification of knowledge that changes its organization, its relative importance, its presentation and its genesis, following the needs of the didactical contract. We called this transformation didactical transposition.

Chevallard (Chevallard, 1999) explains why he maintains the expression 'didactical transposition' instead of calling it, for instance, instructional transfer:

> *(…) it should be clear by now that the proposed rendering means (about) the same as 'didactic transfer' (…) transfer and transposition are both metaphors. (…) as a musical metaphor, transposition may aptly call to mind images closer to what the theory of 'didactical transposition' tells us: Knowledge is not a substance which has to be transferred from one place to another; it is a world of experience which, through a creative process, has to be… transposed, to be adapted to a different 'key'- the child – and to a new 'instrument' in the classroom. (p.7)*

Didactical transposition is a part of the French theoretical work of didactics of mathematics and closely related to the way curriculum is understood. Chevallard (2002) proposes that the curriculum be seen as eight didactical levels, crossing from high generality to high specificity. The levels are:

-2: the level of society,
-1: the level of school,
0: the level of pedagogy,
1: the level of discipline,
2: the level of domain,
3: the level sector,
4: the level of the theme, and
5: the level of subject.

The didactical transposition process consists of the choice made for the content at each level. As I understand the process of a didactical transposition, it is didactical decisions made on different levels and by different people as authorities, researchers, pedagogical consultants or teachers, concerning the content. What to put into the syllabus and how detailed to make it and how to teach different topics, change from country to country. In Chevallard's version, the 'creative process' of the didactical transposition into teaching is not specified.

The analogy between didactical transposition and meta-didactical transposition is however, that the transposition was applied not to mathematics itself, but to the didactics of mathematics. The meta-didactical transposition is a process whereby didactic ideas, from the research literature about mathematics education, are transposed into in-service education. In this study, I practice a meta-didactical transposition when I survey theoretical ideas from such literature and choose what to use, and how to use it during the in-service courses. The content of 'my' courses is didactic of mathematics taught through teaching, and it is not to be found in any syllabus in Denmark, but has to be chosen and transposed by the teacher educator; of course it has to be closely related to the syllabus for the Folkeskole, but it does not have its own syllabus. Below is the model for meta-didactical transposition that I used:

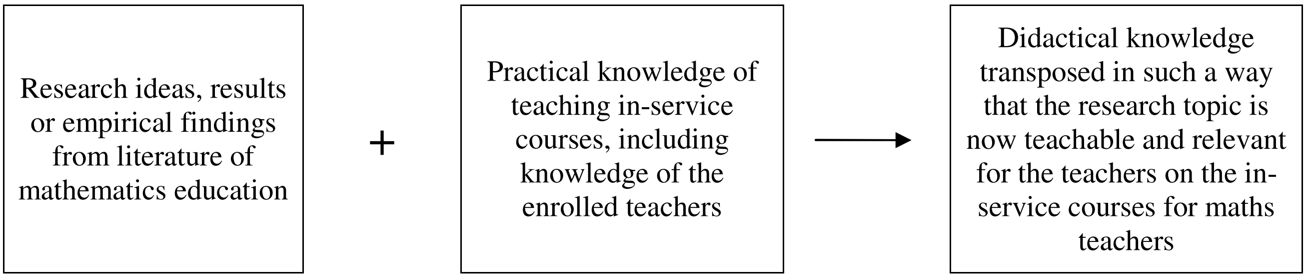


Figure 21: The process of meta-didactical transposition

This process of combining research ideas from didactics of mathematics with practical knowledge from teaching didactics creates the meta-didactical transposition. This transposition is a way to bridge theoretical ideas/results and practice. The difficulties of using theory in practice have often been discussed in research literature (Strauss, 2001) (Rasmussen, 2004). The theories presented to teachers often remain 'just theoretical' for them and are rarely implemented in their practice. My practice in this dissertation is an attempt to base both the methods and the content on research.

1.1 Didactical practice

Creating the redesign called for inspiration in three different respects:

- Goals
- Content
- Form

The goals for the whole course were that the teachers should learn the necessary skills to prepare and run an oral exam in mathematics; this means that they should develop skills for mathematical communication, using open practical problems, not only for the exam but for their daily teaching as well. Preparing an oral exam means to prepare the pupils as well during their daily teaching preferably years before the final exam. For this dissertation, I decided to focus on communication, both mathematical and other relevant communication.

I found the content for teaching people to reflect on communication by a survey in the literature on mathematical educational research; not exclusively for the literature about in-service education but also about teaching mathematics in schools. I had several reasons for seeking out inspiration in the research literature on school mathematics. First, little has been written about how to teach on in-service courses about reflection on communication, most of the research literature concerning communication in mathematics are about classrooms e.g. (Brown, 1997,Alrø and Skovsmose, 2002), and if we look at teaching in general, we find quite a few similarities between teaching in a school class and in-service teaching. The teaching triangle (fig. 22) shows the different roles in teaching: pupil and teacher. To be a pupil or a teacher could be viewed as roles; still such roles are a part of the person. The person in the classroom who is responsible for the teaching and teaches 'becomes' the teacher, and the persons who are taught 'become' pupils. The content is the third parameter in this 'game'. Access to the content differs depending on roles. The teacher chooses and 'controls' it, whereas the pupils are the ones to be made familiar with the content. No matter if we look at teaching in primary or higher secondary, universities or adult education, we find the same roles. The interaction in this ‘game’ is manifested through communication used in the teaching.

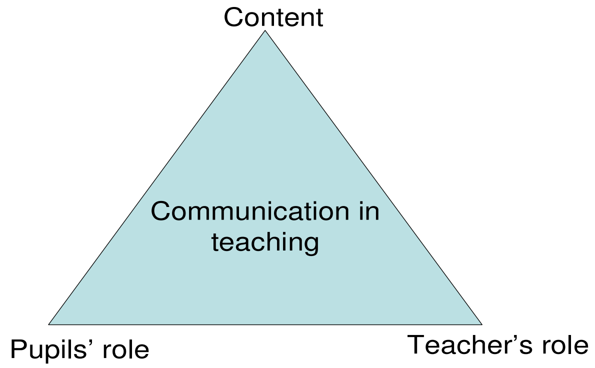


Figure 22: The teaching triangle

I noticed another circular inspiration between in-service and school classrooms: the classroom observations I carried out in the pre-study were meant as an evaluation of C-02, and from that evaluation perspective I realised how the teaching in the school classrooms fed back into the in-service education. The teachers came to the in-service course to be inspired by the teaching in the in-service classroom, and the most important inspiration for the redesign came from school classroom observations I did after C-02.

The triangle with three ‘levels’ shows in-service teaching with teachers as ‘pupils’, and the same teachers as teachers in their own classes (figure 23). We have both similarities and role changing.

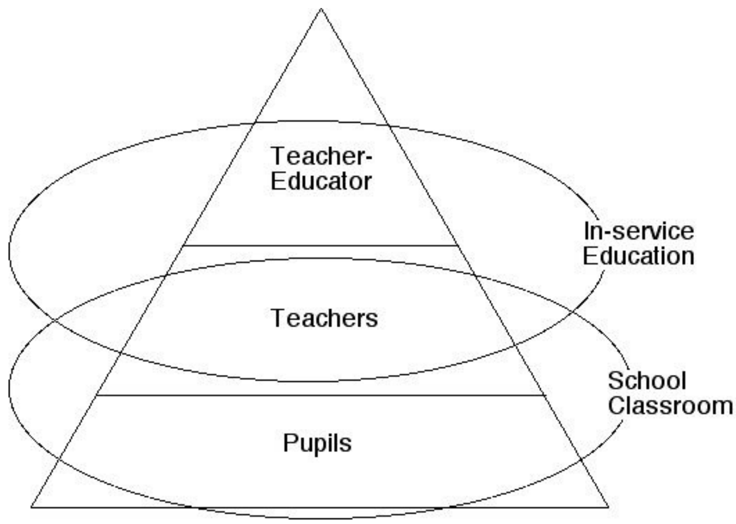


Figure 23: The pyramid of learning and changing from in-service education to school classrooms and vice versa

Figure 23 illustrates, furthermore, how the influence from one teacher educator may be disseminated to many teachers during in-service education, and again to their pupils in schools. I used similarities between all types of classrooms to find appropriate theoretical ideas for use on the in-service courses, and these similarities made me increasingly see the ‘teachers’ as ‘pupils’ and me as their teacher in the context of the in-service training. This insight into similarities made me examine my own practice as a teacher educator. On closer inspection, I became aware of some of my own shortcomings, such as how I forgot to build on the 'teacher's' existing knowledge: when I asked them to reflect on the activities, I just left difficult topics to group work without becoming involved myself or helping the teachers develop their own meta-cognition. What I realised through the classroom observations was that the teaching and the expected outcome did not correlate.

Looking for further similarities between the in-service classroom and the school classroom, I found:

- The concept with a teacher who prepares and runs the teaching for 20-30 pupils (or 'teachers'),
- The organisation of the learning environment in the classroom, which includes the form of the activities, is decided by the teacher/teacher-educator,
- The micro-culture in both classrooms is shaped by dynamic processes and the power of the same 'teaching game',
- Teaching situations are sensitive to a good atmosphere in the classroom,
- The pupils/'teachers' appreciate positive attention from the teacher/teacher-educator,
- Teaching is more effective if the pupil's/'teacher's' 'present knowledge' is known to themselves.

This analogy consists of points and knowledge about teaching, based on my own and other's experiences concerning teaching (Laursen, 2004). As for the atmosphere in the classroom, if it is relaxed, then it is less dangerous to fail or make mistakes, and if it is possible for the teacher/teacher educator to establish individual relations to the pupils/'teachers' that make them feel noticed as unique individuals instead of 'just one in the group', this has a positive influence on the atmosphere in the classroom. Learning is, in one perspective, the cooperation between new knowledge and existing knowledge. Consciousness about this 'cooperation' is what Bruner (Bruner, 1998) p. 125 in the Danish translation) calls meta-cognition, which he explains as: "The learner should be aware of her own thought processes". He also suggests that it is crucial for the researcher and teacher alike to help the pupil to be meta-cognitive.

I mention similarities between the two kinds of classroom teaching because I went looking for arguments for using research meant for the classroom to transpose and use for in-service education; of course I could find lots of differences as well, if I had wanted to.

I took the insight about how teaching and expected outcome did not correlate, combined with the other similarities, into account during the redesign. I decided to create some guidelines for the meta-didactical transposition, which should ensure a larger degree of correlation between the teaching and the expected outcome, and should also serve as a guide for how to plan and run teaching during in-service training. The guiding principles were established on this basis.

1.2 Didactical contract

In this context I will transfer the concept of 'didactical contract' to concern the relationship between the teacher educator and the 'teachers'. Brousseau (Brousseau, 1997) defines the didactical contract with another new term from his didactical universe, a 'didactical situation':

> *The didactical contract is the rule of the game and the strategy of the didactical situation. It is the justification that the teachers has for presenting the situation. (Ibid, p. 31)*

I would, however, rather use the notion of the didactical contract as it is commonly used outside of France:

The didactical contract is the set of the implicit and explicit rules of social and mathematical interaction between the teacher and the students in the particular classroom. (Wedege and Skott, 2006) p. 41)

Both the participating teachers and the teacher educator at the in-service course bring expectations to the course, expectations which could influence the didactical contract during the in-service courses. Formulated in competence terms the goal for the course is:

To qualify mathematics teachers who are required to run an oral examination in mathematics based on the departmental order, and to create a connection between the exam and the day-to-day teaching. (CVU København og Nordsjælland et al., 2002) p. 92) (my translation)

It takes several different competencies to become a good mathematics teacher. Niss and Jensen (Niss and Jensen, 2002) identify six competencies, which should be learned in pre-service education. It concerns the following list of competences:

- To read the curriculum and transform it to teaching
- To plan and carry out the teaching according to the syllabus
- To uncover and interpret the pupils mathematical learning process
- To assess the pupil's mathematical skills
- To cooperate with colleagues and other persons, outside the community, about the frames for the teaching
- To develop and generate personal professional skills as a math teachers

To become qualified to run a final oral examination in mathematics for lower secondary school it is particular an advantage to be a good teacher and furthermore to possess the following list of competences (Ibid), which is the content of the work on the in-service course:

- thinking mathematically,
- formulating mathematics tasks as practical problems,
- modelling mathematically,
- communicating in, with and about mathematics,
- assess the pupils use of reasoning, representations, symbols handling, aids and tools and their oral presentation.

The target group for the in-service course is teachers who are specialised in mathematics, which means that we can surmise that they possess certain mathematical skills. In the pre-study, however, we saw that the teachers lacked mathematical skills, but for such a short course (42 hours) I decided not to teach mathematics specifically, but rather to take it for granted that they mastered the necessary mathematical content, and if this content was a problem for some of the participants, I would recommend them another course. The content was to learn how to formulate practical problems in mathematics and assess the pupils during their work with these tasks. I decided to focus on communication, whether mathematical or not.

1.3 Mathematical communication

The four cases I observed in the pre-study do not display the same difficulties when teaching mathematics with open problems, but for all of them communication and reflection on their own communication were underdeveloped. Overall, the course concerned how to develop competencies to run the final oral exam in mathematics and how to teach the required mathematical skills through open problem-solving. 'Oral mathematics' is closely related to mathematical communication in several ways, such as how the teacher introduces new topics, how the teacher asks questions and respond to questions or problems, how the pupils express their knowledge, how the teacher interpret these expressions, how pupils speak with each other and with the teacher, and finally how the pupils understand what is expected of them. When I analysed the collected data from the case studies, I realised that the teachers were again left with the most difficult tasks to solve on their own: to reflect on their own communication, and to analyse this reflection, and through an evaluation to develop their communication in the classroom to encourage the pupils' learning of mathematics, in other words what Schön (Schön, 1983) calls reflection in and on action and what concerned the professional communication when the topic is learning mathematics through open problems.

This specific study therefore concerns how to teach in such a way that communication is seen as a key factor in the teaching of mathematics and how mathematical communication could be developed through reflection and through an awareness of how to ask powerful questions. The difference between mathematical and non-mathematical communications is both in the content and the form. Mathematical communication requires mathematical skills and a methodology for how to talk about mathematics that has to be acquired; it is not something that is developed in society by itself.

1.4 Summary of 'Meta-didactical transposition'

Meta-didactical transposition is a process through which theoretical ideas or results from the research literature are transformed to fit into teaching at in-service education. The process consists of combining research ideas and results with knowledge about in-service teaching. Research ideas for the meta-didactical transposition could be chosen from ideas or results relating to in-service education or results from mathematics teaching in elementary school. The transposition ensures that the activities are arranged for adults.

The didactical contract comprises the set of implicit and explicit rules of social and mathematical interaction between the teachers and the students. In this study, the contract concerns communication as a key factor in the teaching of mathematics with open problem-solving.

One difference between mathematical communication and non-mathematical communication is that mathematical communication has to be taught for a particular purpose, while other communication is usually learned when being a part of a social community.

2. Guiding principles

The overall issue for making a set of guiding principles was to find a way to connect theory and practice in a synergetic way and transpose them into the in-service course teaching. 'Theory' is to be understood here as research ideas, frameworks or set of results found in research literature. Teachers do not read research articles about mathematics education, and it is not just because most of them are in English, but also because they do not read that kind of things, as they say. One reason might be that they fail to see how it could be useful in their day-to-day teaching; that the effort does not serve a useful purpose.

The set of guiding principles is meant as a kind of rules or scaffold, which I developed from teaching experiences on many levels of teaching combined with theoretical arguments from the pedagogical literature, arguments I will describe in the next part. The guiding principles for the rules are the framework for the meta-didactical transposition. My intention is to investigate how these guiding principles work in practice on in-service courses, in order to see to what extent these principles would influence the teaching and the participating teachers' reactions. In-service teaching that follows these principles will from now on be called 'GP-teaching'; the rules I put up are as follows:

1.
Each teaching sequence on the in-service course is based on a particular theoretical concept, framework or set of results found in research literature.

2.
The theoretical concept is transformed into practical activities for the participating teachers and the activities should accord with the following principles:
   a. teaching objectives make the theoretical concept apparent;
   b. teaching methods are practicable in the teachers' own practice;
   c. activities make the teachers' tacit knowledge apparent;
   d. activities provide feedback mechanisms for reflection.

3.
In the preparation phase, theory precedes the designed activities. In practice, activities are presented before the theory.

These guiding principles ('GP' from now on) represent one way to perform a meta-didactical transposition. The principles are of a form where different content could be used in the transposition. In this study the content is mathematical communication when teaching with open problem solving.

While making the GP and carrying out the GP-teaching, a hypothesis became clear to me: If the teachers became aware of their own (inappropriate) habits and at the same time were offered a theoretical framework that could help them improve that particular habit, this would have a pronounced beneficial effect on the learning processes.

I will present meta-theoretical arguments for each principle and explain how and why each principle plays a role in the meta-didactical transposition. This description will furthermore contain an explanation of how the principles were realised on the in-service course, how the teachers responded, and finally an analysis of how the principles worked at the courses; the rest of this chapter comprises the arguments for the choice of the GP.

The redesign is not a final product, but rather a work in the 'context of discovery'. The redesign consists of processes, where I produce hypotheses, artefacts and new theories about my findings whenever possible. To produce the overall idea of the guiding principles, I needed to use my experiences and knowledge about how 'teachers' learn on in-service courses. The first step in the preparation of the GP was to find useful theoretical ideas from the research to be transferred into mathematical communication; the next step was to argue for the choice, the third to produce practices, and then analyse and evaluate the first redesign so a new cycle of the process could be started.

I searched the research literature, to identify useful theoretical ideas about mathematical communication. I took advantage of the similarities between a school classroom and an in-service classroom. What guided me in the survey was, if I could imagine that I could transfer the concept into activities, through which the 'teachers' could become aware of their own communication. I chose three theoretical ideas from the research literature that were suitable for the GPs. Each of them gave a different, but solid basis for transposition into a didactical design with an emphasis on mathematical communication. My intention and hope was that an experience of these theories would induce the participating 'teachers' to understand the value of developing their professional skills through research findings and articles about communication and reflection.

2.1 GP 1: Teaching based on theory

The first principle is that for each sequence a particular theoretical concept should be chosen. I was inspired to choose research for the first GP by my confusion about the weak expression 'teaching based on research' and the teachers' lack of experience with research literature.

In 1996, when I became a pedagogical consultant at the Royal Danish School of Educational Studies (now the Danish University of Education, DPU), the in-service courses had to be research-based, it was said. 'Research-based' then meant that the teacher educator should be a researcher and the content should be based on research. In the classical interpretation 'research' refers to creation of knowledge that can be generalised and discussed (Schmidt, 2000). Evidence, however, for how the course should be taught to give effective in-service education for the teachers was still unknown. I therefore wanted to find methods to use theoretical research systematically for in-service education and in this sense call the courses 'research-based'.

When I started as a pedagogical consultant, I was not a researcher, but had a lot of experience from being a teacher in more than two decades; at that time I only was half way through my master's degree. The mathematics teachers, who enrolled in 'my' in-service courses, were satisfied with the content and my teaching methods – even though the courses were not research-based in the sense that 'the teacher educator is a researcher'. At that time, I saw it as an example of how the quality of the in-service teaching, measured by the 'teachers'' satisfaction with the course, was not necessarily dependent on 'research-based' or not; in many cases the opposite was the case. Some of the researchers in the mathematics department found it difficult to transpose research into a practice that the teachers found meaningful. It seemed too academic to the teachers, in the sense that they had to transfer the teaching topics into usable teaching themselves. The teachers preferred to have practicable ideas for their teaching directly handed to them.

The in-service education studied in this dissertation is no longer placed within the DPU, but at a 'centre for further education' (called CVU), which is not subject the requirement that the teaching must be 'based on research'; instead it must be 'related to research'. This means that the teacher educator is not expected to be a researcher but the teacher educator must have access to research knowledge in different (unspecified) ways.

Having been a teacher educator for long time, I noticed that when the 'teachers' argued, reflected or evaluated different situations, they rarely used analytical tools. Because I thought it would help the teachers to recognise what they were doing, to be able to see patterns and to express these experiences, I wanted to use research in the in-service courses. I thought it was important for the teachers to know how reflection and assessment are treated in research literature, so that they would become familiar with the discourse. An analytical tool was also necessary for this study. My challenge was how to transpose didactical ideas to make them teachable in such a way that the teachers would be able to understand these ideas and be inspired and able to generate their own ideas in their teaching.

As mentioned before, Danish teachers do not read much specialised literature; there is no tradition for doing so. "Teachers don't read this" was the title of one project (Jørgensen et al., 1992) before the last School Act (1993). It was a comment from a teacher to a researcher, who expressed surprise at how little a report was used. The most obvious consequence is that the teachers do not 'stand on the shoulders of giants', but have to develop their own practice, properly with help from colleagues and the culture of the school, or by using experiences from their own school days.

To be a mathematics teacher in the Folkeskole is to be very practically oriented; the teacher practices 'in private' together with a class. Therefore many teachers come for inspiration for practical tasks when they participate in an in-service course. In my capacity as an external examiner at the final examination of student teachers in mathematics, I noticed that theories from pre-service education often concerned general education, which the students then had to transform into more subject-specific practice themselves. This shortcoming causes a shortage of mathematical educational knowledge when the teacher begins her own teaching in practice. The shift from a student at pre-service studies to a teacher with all the responsibilities and challenges entailed is often called a 'practice shock'. New teachers are insufficiently prepared to bridge the gap between theory and practice - if they have met relevant theory at all. How to use theory in practice is not a developed competence. Shulman (Wood, 2006) claims that 'there is no teacher education' in pre-service education and compares teacher training with other professional education such as Law or Medicine. These educations each have a signature pedagogy, he says, which in this context means 'modes of teaching that have been inextricably identified with preparing people for a particular profession'. Signature pedagogy has three characteristics, Shulman is quoted to claim:

1. It is distinctive to the profession;
2. It is pervasive in the curriculum – there are continuities across the program that are part of what it means to think like a member of that profession, and
3. It is essential as elements of instruction and socialisation.

This description is inspired by the American teacher training system, but still I see similarities with the Danish system. To be specialised into being a mathematics teacher in Denmark is not a socialisation into a profession; rather it contains many different theoretical pieces that is left to the student to synthesise. Each teacher has to find out his own way how to teach, and choose what kind of theory to transfer into practice, if any. In the Danish system, the teacher teaches classes alone, and new teachers do the same job as a more experienced teacher. In such a system with a kind of 'private practice', there is no direct community inspiration among the teachers; the inspiration comes from communicating the experiences. To work in these private practices causes a development of tacit knowledge based on experience without an explicit discourse about being a mathematics teacher. The mathematics textbook has done much of the preparatory work; the rest is methods, activities, and how to present the content. Therefore, when the teachers have to explain their teaching, they tend to express it in what kind of activities the pupils should do instead of what kind of competencies the pupils should learn. A recent Danish investigation of the mathematics teaching in primary schools similarly concludes that mathematics lacks a common discourse (Danmarks Evalueringsinstitut, 2006). The discourse of mathematics communication has to be taught, learned and understood to be effective for generating mathematics teaching.

A useful discourse, as an instrument for developing mathematics teaching discussions, needs analytical tools as inspiration. In my research, I look for theories as tools, which should serve as an analytic tool for the teachers to be aware of communication, reflect upon and perhaps understand it in a new way. This understanding should be based on knowledge from both mathematics and mathematics education, and the teachers should be able to evaluate and choose the 'best professional' discourse in the situation through reflection. It should be clear to the teacher where the theoretical results come from, and how it works. In that way, they will eventually become familiar with research results, which they would not meet on their own. Because of my GP, where I hope that the 'teachers' will not only be content with the way I present the theoretical ideas, but also be able to read more after the course, it would be a great advantage if the theoretical results or ideas were described in a readable article for the teachers. In that way they could keep it for their own later reflection and repetition.

2.1.1 Three theoretical ideas

To find applicable research to meet the requirement in GP 1, I surveyed mathematical research literature for articles concerning communication and reflection. My requirements were that the articles should contain some analytical tools to use for communication, reflection or/and evaluation, in a way that was meaningful to use for the courses.

For the redesign, I chose the following three theoretical concepts:

- A. Sfard and C. Kieran's 'Interaction Flowchart' (Sfard and Kieran, 2001)
- H. Steinbring's 'Epistemological Triangle' (Steinbring, 1998)
- U. Leron and O. Hazzan's 'Virtual Monologue' (Leron and Hazzan, 1997)

I will argue for each of them in the next chapter, where I describe the preparing and the transposition of each idea, why I chose that particular article, how I used it on the course, how I presented it to the teachers, and how the participating teachers reacted into the teaching.

Each of the three theoretical concepts provides an analytical tool for mathematical communication, which is necessary for the transposition. Besides, I managed to find just one particular article about each of the concepts that I could print for the teachers. Thirdly, it was possible to identify one core concept, which I could transpose into an activity for the teachers, and fourthly it was possible to show the participating teachers how they could use the ideas in their own practices.

2.2 GP 2: Practical activities

The practical activities should serve as a vehicle between the theoretical part and the practical application of the theories, in such a way that the theories should become a tool for the users. The second principle comprises four elements, with ‘a’ and ‘b’ related to the theoretical concept whereas ‘c’ and ‘d’ touches on how the activities should affect the participating teachers.

The urge to design activities came from my year-long experiences as a teacher and what I earlier called ‘engaging learning processes’ (p. 86). The main idea in these processes is that the activities provoke some sort of emotions that will give the learners experiences, which could be used for reflection and analysing. Further, the activity and following experience should give insight for the teacher in her own personal (but still professional) difficulties. When teachers are introduced to theory they should be able to find some kind of answers for how to be ‘helped’ if necessary by the theoretical concepts chosen for the sequence.

The idea for the two first elements (a and b) is to find or chose an activity that use the core of the theoretical concept in the transformation and further shows the idea in practice. My argument for this plan is that if I am unable to transpose the idea into practice, how can I expect the teachers to do it later on? Not that it is any official requirement that the teacher educator is able to transform theoretical ideas into a kind of practice, but because the situation on an in-service course is about teaching and I want to act as model, it is important according to my norms that I am able to do it myself. It means that the teaching objectives may show the research idea apparently into practice. As all teaching, it could of course be done in several ways, but I will only argue for the principle itself. To let the teachers do activities at all instead of just listening or discussing is among others inspired from Dewey: “Learn to do by knowing and to know by doing” (Dewey & McLellan, 1889; after (Dysthe, 2003) p. 130). Activities in different communities help shaping meanings. In teaching, this means that beliefs can be shaped in practice without any explicit communication about it, which is also how tacit knowledge is shaped. Therefore it is important to discuss experiences shaped in practice, so it becomes possible to understand how and which competencies should be developed. Furthermore, it is important to experience and discuss what kind of discourse is efficient for teaching discussions.

The teaching triangle consists of a ‘teacher’, ‘pupils’ and ‘content’. The content can be many different things, and likewise the ways to prepare and organise it for teaching and learning. I will describe three different ways based on different positions (Hviid, 2003): one is where the teacher educator are active in delivering and the ‘teachers’ are active in their receiving; this way is called ‘applicative learning conception’. In this conception, the ‘teachers’ are thought of as uninformed; another method consists of playing out the teaching-learning scenario as a process where the ‘teachers’ and the teacher educator are equal partners, where both are expected to influence what happens, and where the power and responsibility are shared between the partners. It means that the content is negotiated; this is called ‘integrative teaching practice’. A third way is to look at the content and investigate the different ways the teacher educator and the ‘teachers’ understand the content. If the teacher educator and the ‘teachers’ had the same knowledge and understanding, the teaching would perhaps be needless. In this third way, the learning process is based on how the learner develop new competencies based on knowledge and understanding of the content and where the teacher educator’s work is to shape and navigate in a potential zone for learning, what Spillane (1999) calls the ‘zone of enactment’.

I prefer the third way. I understand the interaction between the ‘teachers’ and the teacher educator as a ‘play’ among different partners with different forces (Ejersbo and Michelsen, 2005). The teacher educator has the expertise to access research knowledge, while the teachers are experts on their own class practice with their own practice-theory. At in-service education, the teacher educator – with her practice-theory – is responsible for presenting the content and organising the methods to be used, whereas the participating teachers are responsible for being active and ready for a learning process. They must accept and take part in the prepared activities and be willing to reflect on and work with the activities. The didactical contract must contain trust from both the teacher educator and the 'teachers' about what will happen in the teaching/learning situation.

As a teacher educator, I adopt three distinct but interrelated teacher roles: One is as an organiser and starter of activities, and it is my responsibility to assist the learners in becoming aware of their preferences; in this situation, I observe and get involved as necessary. Another role is as listener and discussion partner, when I evaluate their presentations and give feedback on their works through encouraging critical questioning. The third role is when I present new knowledge, which I often do as a kind of lecture; in this kind of lectures questions are allowed and welcome during my presentation.

In short, the aim of the activities is to create a situation where the theoretical concept, the content of the activity, becomes visible and meaningful to the teachers, who furthermore, through the activities, become aware of their own difficulties/habits related to the same areas through reflection.

2.2.1 GP 2, a and b: The teaching objectives and methods

The two principles 'a' and 'b' describe how the theoretical concept could be transformed into practice in such a way that a practical value of the concept becomes apparent that the teachers can be inspired to do something similar in their own classes. It means that the main idea in each theoretical framework should be taught in such a way that the 'teachers' can experience, emotionally and analytically, how the theoretical concept can work in practice. The intention is to show what the theoretical concept looks like in a practical context.

In-service education concerns learning, which I will describe shortly and briefly here, because I later have more detailed explanations of learning and reflection combined. Learning can be understood as a 'process of changing capacity'. It means that during the process, the learner develops several competencies. Piaget (Piaget, 1973) distinguishes between different ways of learning. He describes invariable processes as an endeavour after equilibrium with the surrounding environment, where the process goes both ways as interplay between the person and the environment. This adaptation process consists of two functions: assimilation, which is to get new ideas to fit into old patterns, and accommodation, which is to reconstruct old patterns to fit with the new ideas. In my study, the teachers will be presented with new ideas, which will maybe fit into their old habits or maybe they will have to change some beliefs and habits during their learning processes. It depends on how 'disturbed' they will be. If the teachers' expectations are more or less fulfilled, the expectation will match the teachers' hypothesis, and the teachers will assimilate the ideas, but if the teachers meet teaching that lies outside their expectation, they will perhaps experience a mismatch, which can provoke an adaptation in a form of accommodation.

How we learn and when we get 'disturbed' is connected to how we understand the reality we act in. As I understand the process of assimilating or accommodating knowledge, I think that the knowledge has to be in a form that fits the perceived reality. Learning happens with a reference to the expectations that the person already has established. It means that we construct how we see the world to understand our own experiences (Glaserfeld, 1995) p. 63). For the teachers it indicates that they will always use their old experiences to adapt to new knowledge. Assimilation is easier to deal with than accommodation, which means that it is easier to enlarge an existing knowledge instead of a deeper reorganising, where it is necessary to build new knowledge. Therefore it is important to work with how it is possible to help accommodation.

With the transposition of the idea into a practical teaching, I try to establish an environment which could be a kind of ‘community of practice’ (Wenger, 1998,Lave and Wenger, 1991) or an analogy to this term. Wenger writes that learning is about acquiring competencies related to what is valued in the community we are a part of. Through this participation we get our identity and produce meaning. In this interpretation, learning is seen as social participation, as being engaged in action, as developing identity, and as shaping meaning through experiences. The participating group of ‘teachers’ at the in-service course can be see as an analogy to a ‘community of practice’, if they learn from doing activities valued in this group, and if they develop their identity and shape meaning through learning these valued competencies. For the individual, a community of practice means to engage and to contribute to the community, to be open for learning to take place. The community becomes a medium, which the teachers can learn through.

With this in mind, the cooperation activities prepared for the in-service courses should so to speak ‘walk the talk’ and let the goal be apparent through the methods, which could be achieved by creating an environment where the teachers experienced the core idea of the theoretical concept through participation.

Furthermore, the teaching methods should be practicable in the teachers’ own practice. This means that the teacher-educator has to teach in such a way that the ‘teachers’ can recognise the idea and experience how the theoretical framework could work in practice; and maybe even be inspired to experiment with the same kind of teaching themselves. As I described earlier, the similarities between the two classrooms – schools and in-service – made it possible to deliver more or less similar teaching. One of the teachers, Tina, said in her interview “How should I know what to do?” which indicate that she lack knowledge about different ways of teaching. Another one of the teachers, John, said that he did as his old teacher did; I often hear that from pre-service students, when I am an external examiner for mathematics examination. The ‘teachers’ had indirectly learned how to teach through years of participating in some kind of classroom life and often they do what their old teachers did; it is a part of their experience. Teaching is a cultural activity and classroom discourse is highly socialised and almost automatic, and that is why it is so difficult to change (Stiegler and Hiebert, 1999), (Stiegler and Hiebert, 2004). We do what we have been able to learn so far. Therefore it is important to present new teaching ways when we teach, and not just talk about it, but also show it.

To change habits is difficult, but the process could be helped if the ‘teachers’ get access to what could be replaced instead of old inconvenient habits. If the teaching on the course could be an example in such a way that the presented theoretical concept works in practice, it would be trustworthy, and trust is crucial for our learning process (Cummins, 2000). Cummins describes the importance of trust and how the feeling of being cheated destroys this trust; these feelings are developed from an early age and are a part of being human. Just telling the teachers that they should transfer the theories into practice, and later on being surprised that it did not happen, is to relegate the most difficult aspect to a lower level in the education hierarchy, and I see this as a way to cheat the teachers. To show them how this could be realised, and perhaps to discuss on a meta-level how it was done and how it felt, is trust-inspiring, and in that way the ‘practicable’ principle is important.

2.2.2 GP 2 c and d: The activities’ influence on the teachers

The two elements c and d concerns requirements to the activities, which should activate the teachers in such a way that their habits and tacit knowledge become apparent to them, and should furthermore, provide a feedback mechanism for reflection. I use the term habits and tacit knowledge: by habits I mean the set of routines we develop when we do the same things many times, such as teaching the same classes, and which we gradually learn to do with little or no mental effort; tacit knowledge includes these habits, but also beliefs and values, which are things that are developed in the culture. I will later explain how I see habits and tacit knowledge influence our decisions and acting.

To respect the principles, the activity should be organised in a way that the teachers' reflection on their own reactions should be a natural and organised part of the program. The intention with the teaching activities is that the activities require that the teachers have to use their mathematical and pedagogical skills in the work. Hopefully during the processes, some of their own teaching habits and mathematical skills will become apparent to them, so it becomes possible to discuss and reflect upon them.

Reflection on communication is one of the main topics in this research, but reflection and communication are not unambiguous concepts. I wrote about communication and mathematical communication in previous chapters, and in the following, I shall develop my version of reflection, which is often used uncritically as a vague theoretical concept. Different definitions of how reflection is conceived and how to use it show us the different facets of the word. I chose the following explanations of reflection because they inspired me to see the concept multifaceted and helped me to see what kind of vehicle it is in this context.

Wahlgren (Wahlgren et al., 2002) p. 17) writes a general explanation about reflection:

> *Reflection is more or less conscious and includes more or less consideration of the connection between our actions and its causalities. Through reflection we will be able to make out what to do in a certain situation, if we want to achieve our goal.*

This answer shows that reflection here is conceived as a tool for improvement of action.

Schön (Schön, 1983) (Danish version 2001) writes about different kind of reflection; I will merely mention 'reflection-in-action' and 'reflection-on-action'. Reflection-on-action is a way to work with the experiences after the action, when the teacher prepares the next lesson and take into account how the previous lecture went, while reflection-in-action is connected to the immediate reaction in action where the teacher reacts in a reflective way to the unexpected. The adjustment takes place in action based on reflection processes. The theoretical framework of reflection in/on action is based on empirical studies with examples from work places. One of Schön's examples concerns a teacher, who, after a lesson that was videotaped, examined two of his pupils on video. The two pupils played the game 'copy my pattern' where they were only allowed to explain their pattern made of coloured pieces of wood without showing it to the one who should copy it. One of the pupils had trouble and the teacher was convinced that this pupil was unable to understand (read: the pupil was too stupid) what the first pupil said, but on closer inspection it turned out that the first pupil told the 'stupid' pupil to take a green triangle, which did not exist in the game, and the 'stupid' pupil's solution was, with this information, not so stupid after all (Ibid p. 85). When the teacher saw the video, he understood that he did not notice in the situation why the 'stupid' pupil had trouble, but when he saw the video, he understood the cause. The conclusion was that reflection-in-action did not grasp the details which reflection-on-action did, particularly because the situation was saved on video. But Schön still concludes that reflection-on-action can develop reflection-in-action if the teacher's reflection is focused on how meaning is understood and decisions are made in actions.

Fibæk Laursen (Laursen, 1997) p. 62) defines reflection as consideration and thoughts, and 'reflexiveness' as a special way to reflect upon one-self. Educational 'reflexiveness' means reflection upon the ideas, their status, basis and possibilities to influence the teaching practice, he writes. Furthermore, he points out that it has become more common and necessary to justify one's teaching, if critical questions are asked by pupils, parents or colleagues. It forces reflection upon one's own teaching to find out if it should be changed. The way to develop is through a critical analysis of the teacher's own practice and of one's beliefs about how teaching should be, he explains.

In my teaching activities, I need reflection as a vehicle for the teachers to become aware of their teaching habits and automatics tacit knowledge. Therefore I consider reflection as consideration about one's tacit habits and automatic actions in such a way that the interplay between actions and causalities becomes apparent, insofar as it is possible.

Reflective action/thinking and automatic actions are considered two distinct kinds of reasoning (Evans, 2003). The idea, which in recent years has increasingly been accepted by the research community, is that there are two quite separate cognitive systems underlying thinking and reasoning, and that they have distinct evolutionary histories. The two are denoted 'System 1' (S1) and 'System 2' (S2) by Stanovich and West (Stanovich Keith E. and West, 2003).

> *System 1 is generally described as a form of universal cognition shared between humans and animals […] it is a set of sub-systems that operate with some autonomy. […] System 2 is believed to have evolved much more recently and is thought by most theorists to be uniquely human. (Evans, 2003)*

The S1 processes are most often described as formed by associative learning processes of the kind produced by neural networks; while S2 permits abstract hypothetical thinking that cannot be achieved by S1. As for an overview, I use a model of the Dual-Process-Theory (DPT):

System 1 (S1)	System 2 (S2)
Non-conscious	Conscious
Automatic	Controlled
Inflexible	Flexible
Fast and effortless	Slow and effortful
Associative and/or heuristic	Rule-based
Parallel	Serial
Nonverbal	Language-involving
Heavily constrained by biology and not directly responsive to verbal instruction	Shaped by culture and directly responsive to verbal instruction
Highly contextualized	Decontextualised

Figure 24: The 'Dual-Process-Theory' (DPT)

S1 processes are characterised as being fast and automatic, while S2 in contrast are slow and conscious.

Dreyfus and Dreyfus (Dreyfus, 1999) describe in their learning theory five steps from being a novice to be an expert. They claim that as we progress from a novice to expert, reflection decreases as the person develops the competence; this means that for an expert, decisions are often based on intuition and experience. When a novice learns and develops the actual skills, it is appropriate that the person makes use of reflection in the learning process to decide on a strategy. The examples given by Dreyfus and Dreyfus are both practical skills as typing on a computer, playing a violin, and intellectual skills such as doing mathematics. But Dreyfus and Dreyfus only explain a part of the results of being 'an expert': they explain how to solve a problem change from being problematic to be solved easy based on experiences (S1). Their definition of an expert is narrow and explains only about mastering a cluster of skills. Yet, new problems will always arrive for the real experts and how to solve them characterise the individual expert. Jarvis (Jarvis, 1999) p. 54) writes as well about the processes we go through to habituate our practice. Based on Dreyfus' five steps, he develops further a five-stage process through which our actions become habituated:

Experimental or creative action; these forms of action might be a slight adaptation to previously enacted behaviour, or they might be entirely new.

Repetitive behaviour; we try to repeat what we have learned of performing and act in precisely the same way.

Presumptive action; we act unthinkingly in a situation as if it were instinctive. This presumptive position is habituation.

Ritualism; we do not longer have to think about the situation, we might miss little differences that tell us the situation has changed.

Alienation; we act in a conforming manner but without meaning because we are powerless to change our behaviour.

As an example of how dangerous it could be to ritualise actions, he tells a story of a doctor with an influenza patient. The doctor has been called out to so many patients with influenza that he does not listen carefully to the symptoms before he writes a prescription. Unlike Dreyfus, Jarvis does not talk about experts, rather he writes:

> *What makes experts experts is that they problematize their situations: they keep learning, even when it is easier to habituate and not learn. Expertise does not come naturally; it is a discipline of continually seeking improvement, which can require a great deal of effort. The experts are always operating in an experimental mode, even though their experience makes work situation seem simple, and they adjust to changing circumstances with apparent ease. (Ibid, p. 55)*

If we compare 'ritualism' and 'alienation' with S1 processes in DPT, we find that when acting is habituated it becomes automatical and unconscious. It means that habits based on experience are both automatic and works in the 'S1' way; they become a part of a person's tacit knowledge. As for the teacher, he develops tacit knowledge in his practice and behaves with habituation (Skott, 2000).

Even though the habituation is inappropriate, it is difficult to change it. It needs incentives because we are usually not aware of most of these habits. As we see, habitual actions could be placed in S1 and reflective actions in S2. During the process of habituation, S2 processes gradually become S1 processes as they become more and more automatic, and it becomes difficult to reflect on these processes; they become sub-conscious. To reflect on, and to be aware of why we do things the way we do, is quite another process, which is interesting for the course.

To look further into how we can use reflection as a vehicle for understanding how our beliefs and actions play together, I will refer to Mezirow.

In his theoretical work, Mezirow (Mezirow, 1990) distinguishes between reflective and non-reflective actions. Reflective actions refer to critical reflection of the circumstances for the problem/object for thinking or habits/automatic reactions. He makes use of the expression 'frames of reference' to refer to the way critical reflection triggers transformative learning. Mezirow views learning as a process based on criticism of prior interpretation in order to construct a new or revised interpretation of meaning. One's critical reflections on experiences thus act as a guide for future action. Our beliefs and the way in which we make meaning are closely related, but neither needs to be encoded in words. Mezirow writes:

> *Transformative learning refers to the process by which we transform our taken-for-granted frames of reference (meaning perspectives, habits of mind, mind-sets) to make them more inclusive, discriminating, open, emotionally capable of change, and reflective so that they may generate beliefs and opinions that will prove more true or justified to guide action. (ibid, p. 8)*

When teachers act in such a way that their 'habits of mind' will be apparent to them, maybe not in words in the first place, but perhaps more like a feeling, it will be possible to work with these habits in a reflective way. For adults, learning depends both on experiences and on habits, which creates the framework for how meaning is understood. The teachers' expectations of in-service courses come from these experiences and habits; their experiences set the limits for the expectation. Mezirow (ibid) differentiates between two dimensions when one creates meaning: the first one is 'Meaning-forming', the activity by which we shape a coherent meaning and implicit habits or expectations; the other one is 'Reforming of our meaning-forming', which is inherently epistemological and takes into account the role of hypotheses and the way interpretations are made. These processes are informed by the culture in which we are embedded; what we learn and do depends on our 'meaning-forming' and our ability to reform our meaning-forming, he says. In cases where reflection acts as a vehicle for reforming old habits by creating new meaning-forming, Mezirow talks about transformative learning. This way of working, however, calls for a need to know how to reflect, and a context in which to reflect on unconscious habits. As far as Mezirow is concerned, the form of 'reflective discourse' (community of understanding that involves shared commitments) is the primary form through which transformative learning takes place. Fostering transformative learning involves helping learners bring the sources, nature, and consequences of taken-for-granted assumptions into critical awareness so that appropriate action can be taken. In this context, the teachers' reflection is used to start a transformative learning process.

Reflection or observation of the second order means that the observer is aware of the position from where the observations are made (Luhmann, 2000). This kind of reflection corresponds to the ability to look and maybe understand from other points of view than the ones a person normally uses. One method to help changing the viewing position is called working with 'Reflective teams' (Andersen, 2005). In his book Andersen describes the method with changing roles of who observes whom and what is to be discussed, how new knowledge is provoked and how it shaped other kinds of positive respect among those involved. 'Reflective teams' are already used in many schools because teachers are generally familiar with this method; in the yearly catalogue from CVU, many teachers enrol in courses, in which working with 'Reflective teams' is offered. The roles and rules are often pre-determined in such kind of work. Preparing the activities for the in-services courses, I planned different role-plays inspired by the thoughts behind 'Reflective teams'. My reason for using this theory is that I wanted to give the teachers an opportunity to be aware of and capable of reflecting upon their implicit assumptions, and possibly give them the appropriate 'tools' to enable them to react when needed. I found that the methods used to attain this goal could be taught through role-plays. When the 'teachers' solve mathematical tasks, they are 'pupils', or at least it feels like that for most of them. When other 'teachers' play the role of observers, they have an opportunity to have their observations discussed by changing position in the reflective teams; it is crucial that the 'teachers' assume the role they are asked to assume. The learning processes are intended not only to engage the participants, but also to create situations that can work as a feedback mechanism for reflection. I found that it was sometimes difficult to transfer the experiences from the role-plays if the reflection was 'outside' the role-playing; therefore I wanted it to be a part of the play. Henriksen (Henriksen T.D., 2006) writes about his research into learning game processes as a development from 'tools of entertainment' to 'tools of learning' and about how this only happens if it is possible to transfer the experiences that spring from role-plays to other situations outside the game as well. The challenge posed by this approach, as I see it, is to find methods to create this feedback. Role-playing or changing positions inspired by the principle underlying reflective teams are used and could be one method, if it is possible to transfer the experiences from the role-playing into learning tools. This will be investigated in the analyses.

In short, I understand reflection in this context as: To be aware and critical of one's actions, both the automatic and the habitual ones, in such a way that it becomes clear how the connection between actions and causalities is structured. The core idea in setting up these activities is to use reflection as a vehicle for transformative learning. Furthermore, the aim is through processes consisting of role-playing to make automatic habits visible in such a way that the teachers become aware of them, identify any problems, and from that point discuss and reflect upon how to solve these problems. The activities will hopefully engender a motivation for learning how to solve these problems through research ideas or results, which will be presented to the 'teachers' through lectures in the last step of the workshops.

2.3 GP 3: The order of activities and theory

This principle is connected to the old question whether we should know about things or phenomena from 'books' before we acquire practical experiences, or whether we should experience things before we search for knowledge in books that explains the phenomena.

I apply two rules to this third principle. The first is that the research concept should be chosen before any activities are planned. My arguments for this are mostly based on my own experiences. Dewey's activity pedagogy 'Learning by doing' has influenced many teachers in the past three decades, where I have worked in and with schools. Very often, I have seen how different projects, which the pupils liked very much, were more aimed at the activities than at the learning objectives, and at the end of these projects, the teachers were often surprised that the pupils did not learn what the teacher had expected. Using activities requires that the activities are chosen as a vehicle for the learning goals. If the goals are formulated and thought of as competencies, both intellectual and practical, is it possible to investigate whether the learning goals is reached, because competencies are related to how people act. One of the difficulties is to find a suitable activity that would fit the learning goals and, in this study, the theoretical concepts. Therefore the activity in itself should be one of these theoretical concepts transformed into practice. I admit that it is easier to find 'good' activities that resemble playing for activating pupils without thinking about what the aims of the activities are, but to do so often imply that the activities and the learning goals do not interrelate.

The other rule is that the activities are presented before the theoretical part. The reason is my hypothesis that the teachers will see the meaning and feel a need to learn the theory better when the knowledge is conveyed as a personal experience before the more abstract theoretical concepts are presented. When the knowledge is activated through an activity the teacher has just take part in, it is possible that the teacher will activate existing personal knowledge. The didactical contract is not ‘open’ in the sense that the ‘teachers’ know my plan for the activity before they start, but this is a part of the plan. An analogy with this order of ‘experiment before explanation’ is a classical example of a geometric puzzle (invented by Brousseau) the goal of which is to develop the pupils’ knowledge about proportions. The pupils in a group get a puzzle, where the pieces are triangles and quadrangles, as in the example:

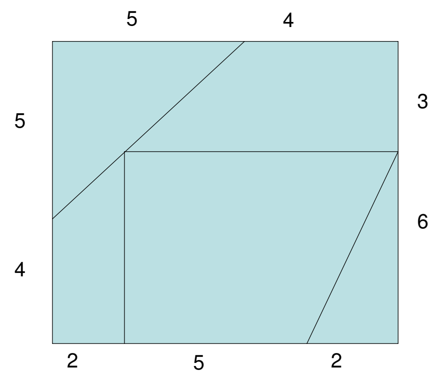


Figure 24: A puzzle

The task for the pupils is to make a bigger model of the puzzle; the part that measures 4 should measure 7 in a bigger model. Each pupil has to enlarge one piece. The point is that if one makes a mistake, the pieces do not fit together; all the pupils have to understand how it works. If the teacher explains proportions to the pupils before the pupils work with the task themselves, the process will be more likely to reproduce the knowledge, but if the pupils are allowed to try it out before they hear the explanation, they may have a personalised experience with the task before the ‘official’ mathematical explanation is offered (the specific task is from (Winsløw, 2006) p. 69). It works in the same way with the didactical ideas that I present to the ‘teachers’ on the course. This priority implies that the ‘teachers’ are more likely to trust that the activity is worth doing.

We know from other research projects that teachers do not read much about theoretical research. I therefore decided to try out if this order of things would make any difference for the teachers’ inclination to be interested in relevant research and to eventually read the articles the source of the theoretical concept, or if they just preferred to hear my explanation. The choice of order in practice thus is containing an experiment, which it is possible to investigate by interviews of the participating teachers. All the articles are in English, which I knew would be a hurdle for the ‘teachers’.

2.4 Summary of ‘Guiding principles’

The guiding principles form a framework for the meta-didactical transposition:

1.
Each teaching sequence at the in-service course is based on a particular theoretical concept.

2.
The theoretical concept is transformed into practical activities for participating teachers and the activities should accord with the following principles:

   a. teaching objectives make the theoretical ideas apparent;
   b. teaching methods are practicable in the teachers’ own practice;

c. activities make the teachers' tacit knowledge apparent;
d. activities provide feedback mechanisms for reflection.

3.

In the preparation phase, theory precedes the designed activities. In practice, activities are presented before the theory.

These principles are used to transpose the following three theoretical concepts into in-service teaching:

- A. Sfard and C. Kieran's 'Interaction Flowchart' (Sfard and Kieran, 2001)
- H. Steinbring's 'Epistemological Triangle' (Steinbring, 1998,Steinbring, 2005)
- U. Leron and O. Hazzan's 'Virtual Monologue' (Leron and Hazzan, 1997)

3. Redesigns

The redesign consists of the guiding principles (GP) and the content for each of the workshops about communication is guided by them. Other things still take place on the course and are not necessarily guided by these principles. The way the content was presented to the teachers, together with the activities, has been improved during the courses from C-04 to C-06. Furthermore, a new theoretical framework has found its way into the courses. As for an overview:

C-04: The three theoretical ideas, A. Sfard and C. Kieran's 'Interaction Flowchart' (Sfard and Kieran, 2001), H. Steinbring's 'Epistemological Triangle' (Steinbring, 1998,Steinbring, 2005) and U. Leron and O. Hazzan's 'Virtual Monologue' (Leron and Hazzan, 1997) were tried out in accordance with the GP. From these workshops, I will describe how the GP worked and present a more detailed description from the different workshops as documented by my own log and the teachers' logs.

C-05: The same three theoretical ideas were applied, this time in an improved form based on experiences from C-04. From C-05, I will describe the improvements, the activities and the presentations. Furthermore, I will describe and document a situation from the workshop where the participating teachers work with the activities, designed with a form of the 'Interaction Flowchart' and the 'Epistemological Triangle'. The underlying documentation is in the form of videotapes. A new theoretical concept, 'Sociomathematical norms' (Yackel and Cobb, 1996) was tried out, because I lacked a tool that could help the teachers when they were having a class communication about mathematical results obtained by the pupils. This case will be described briefly.

C-06: The four concepts were repeated with the necessary improvements in both activities and presentations. Documentation is in form of materials produced by the 'teachers' and the materials produced by me, including logs.

The teachers were not subsequently observed in their classes in this part of the study. The results are based only upon the recorded reactions of the participating teachers during the courses, and do not include whether they made a new didactical transposition in their own classes after the in-service course.

Following the GP, the first task was to identify viable theoretical concepts. My choice for articles could have included other articles, but for the experiments and testing of the hypothesis underlying the guiding principles I chose the three concepts mentioned above. The presentations of the three concepts was done separately for the Virtual Monologue (Leron & Hazzan, 1997), while Sfard and Kieran's Interactive Flowchart (2001) and Steinbring's Epistemological Triangle (1998) were presented and used in the same workshop to supplement each other.

The next step in the redesign was to design a meta-didactical transformation of each of the theoretical frameworks into a tractable activity. The task was to find activities and to use open tasks at the same time. A closer description of this work can be found in the chapter: 'Preparing the course'.

A presentation of each theoretical concept was to follow after the activity. The following presentations of the 'Interaction Flowchart' and the 'Epistemological Triangle' are what I chose to pick from the articles and later from the book: "The Construction of New Knowledge in Classroom Interaction" (Steinbring, 2005). I organised these parts as lectures to the teachers in continuation to their practical activities in the workshops. I used OHP sheets in my presentation to the teachers.

3.1 The Interaction Flowchart and the Epistemological Triangle

Interaction Flowchart: 'Interaction Flowchart' (IF) is an analytical tool to get systematic data out of observing mathematics communication in the classroom. The main idea of using IF is to use defined symbols that show who is speaking to whom, if the particular utterance is pro- or re-active and the level of the content the utterance contains, categorised as object-level, meta-level or unclear nature. IF was developed by the authors (Sfard and Kieran, 2001) as a 'pre-occupation', which in this context means a meta-level where it is not just the content of the utterances that are interesting, but also the pattern who is pro-active and who is re-active during the communication. The use of IF works as an eye-opener to see miscommunication, which again shapes a need for an additional way of understanding and looking at communication interaction in mathematics teaching. In this context pro-active means that the source utterance invites a response, so that the following utterance is an expected reaction, while re-active means that the source utterance is a reaction to the target utterance. The analysis involved communication between pupils who solve mathematical problems or between the teacher and some pupils who solve maths problems. The analysis is elaborated to prepare teachers with a useful tool in their role in the mathematics classroom where 'teaching mathematics through conversation' is recommended. To work with mathematics teaching through conversation is one way to practice the requirements for not only the final Danish oral exam in mathematics, but also the requirement of one of the points in the Danish syllabus for elementary school, called 'communication and problem solving'. The authors' intention with developing the IF is to make it an instrument with which the teachers would be able to monitor, help and regulate their own and their students' future activities to be more effective in the 'talking classroom'. They express it in this way:

> *Preoccupation analysis deals with the question of how the participants of the conversation move between different channels of communication (private, interpersonal) and different levels (object-level and meta-level). Our principal tool in this analysis is the interactivity flowchart. With the help of this special instrument one is able to evaluate the interest in activating different channels and in creating a real dialogue with their partners. (ibid, p.192)*

The IF tool includes arrow symbols for the single utterance and each arrow symbol has a special meaning:

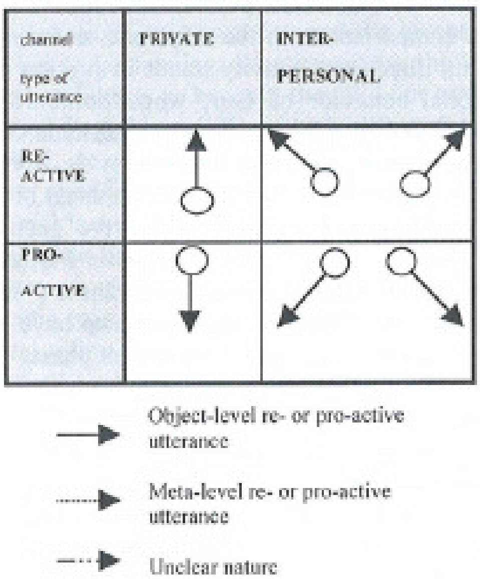


Figure 26: Interaction flowchart symbols (ibid, p. 195)

In the article, a conversation between to pupils is analysed with the IF- model. Sfard & Kieran explain how their model helps analysing the complex interpersonal communication:

At any given moment, each participant is simultaneously involved in a number of object-level and meta-level activities: In trying to understand the explicit contents of previous utterances and to produce new ones, in monitoring the interaction, in presenting herself to others the way she would like to be seen, in engineering her position within the given group, and so on. (…) As a discourse evolves, participants' attention is moving between channels – between one's own line of thought and those of his or her partners and it also winds back and forth between the explicit object of discourse and meta-discursive considerations. All this is true whether we speak about communication between students or between students and teachers.

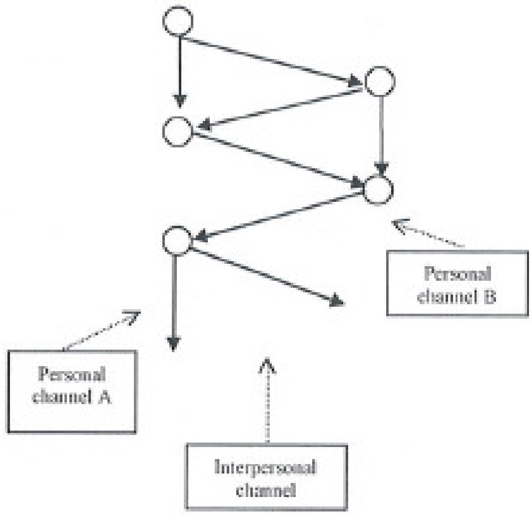


Figure 27: Dialogue as multi-channel communication (ibid, p.192)

The model shows the communication between two pupils, and the arrows tell us that the pupils utterances are both private pro-active and interpersonal pro-active and the content is on the object-level. The interpretation of the communication between the two pupils in the articles showed with different arrows a pattern in the pupils' communication, where it became clear that the two pupils never really solved the given problem together. They had different ways of working and understanding, and the IF tools showed that they never resolved these differences. In fact it became apparent that their different pattern undermined the effectiveness of the pupils' interaction, rather than bringing any real improvement.

The 'IF system' could be a useful tool related to understand communication; how and when the mathematical communication helps bring about a shared understanding of a concepts and solve tasks, and when it does not. One weakness of the IF is that the focus is more on the interpretation of the interaction between pupils than on the utterances about mathematics. The IF-system alone was therefore not enough for a useful tool to observe mathematics development for the individual pupil.

I chose to use IF anyway, because I see it as a useful analytical tool for teachers who observe communication, both mathematical and non-mathematical. I hoped the work with this focused observation model would develop the teachers' awareness of communication in the classroom and especially how to help pupils develop their communication about mathematics. At the course, it was used among the teachers when they solved mathematical problems; the details about this use will be presented later.

The Epistemological Triangle: The epistemological triangle (ET) is another analytical instrument. ET is defined as:

> *"…an instrument of analysis, which tries to identify different kinds of relationships that are constructed during the classroom discourse between signs or symbols and contexts of reference" (Steinbring, 1998, p.112)*

ET is a theoretical framework developed by H. Steinbring, and it is described in a number of articles and books. The framework is a philosophical and epistemological critique of the belief that learning and teaching mathematics can be organized by developing and optimising a technical mathematical language and terminology – which the German 'stoffdidaktik' is based on, he says. ET is based on the idea that mathematics is not a simple image of the physical world, but consists of concepts that undergo developments and changes, which cannot be conceived of as a technical language or terminology:

> *Mathematics must be seen as a vivid, open language and a means of communication that produces its own metaphors, meanings, and interpretation. (ibid, p.106)*
> …
> *Thus, the epistemological perspective on classroom discourse means looking for the specific semantics of mathematical knowledge that is constituted in the interactive process and in what way the constituted meaning matches a rich semantic structure of theoretical mathematics knowledge. (…) The epistemology of mathematics claims that theoretical meaning also exceeds actual social practice and thus 'exists' outside the borders of the already known social habits and practice.*

This quote, in relation to the didactical transposition, touches on what goes on in the epistemology of mathematics, on how the concepts are understood and learned. The mathematical communication going on the classroom must, according to the ET, be analysed in the way it is used together with the signs which represent it and the concepts it is related to. Steinbring uses Luhmann's view of communication, which comprises three selections; selection of information, selection of communication form, and selection of understanding (Luhmann, 2000). In this interpretation, communication as transfer of understanding is not plausible; what is understood by the receiver(s) or interpreter(s) always depends on the interpretation. Understanding is created by the psychological system of the interpreter. The consequence of this view for mathematics education is

> *… that direct connection or immediate influences between the social system 'mathematics teaching' and the psychological system 'mathematical learning' of the students are definitely impossible. (…) Therefore teaching cannot automatically induce understanding in the consciousness of students. (Steinbring 1998, p.388)*

The ET system builds on the relationship between entities, objects and signs. In a semiotic understanding, a (mathematical) sign represents something else, while it means understanding in the frame of the epistemological interpretation of mathematics knowledge (Steinbring, 2006). In order to code and register the knowledge, certain signs or symbol systems are required when we work with mathematics. These signs have a mathematical meaning, but not to the pupils, before meaning is produced by the epistemological mediation within a suitable relevant context. The triangular connecting scheme between the mathematical sign, the context and the mediation between signs and reference context, which is influenced by the epistemological condition of mathematical knowledge, can be represented in the ET (Steinbring, 2005):

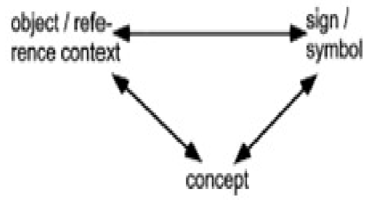


Figure 28: The epistemological triangle (Steinbring, 2005 p. 22)

The following example is from teaching in lower primary classes where real world problems and pictures often are chosen as reference context:

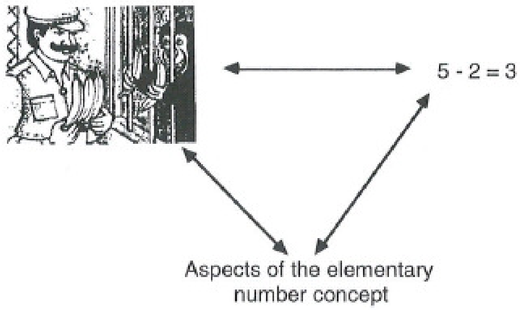


Figure 29: The epistemological triangle with an empirical reference context (ibid, p. 25)

I chose this example with the picture, because I realised that it helped the 'teachers' to understand how an ET could be completed. The lecturing about the ET was done after the workshop and therefore I could also use several of the reactors' observations and discuss how they used the ET therein. Unlike the other research ideas, the interaction flowchart and the virtual monologue (see later), the ET is a theoretical framework, which means that it is a theory about the way mathematical concepts are developed from epistemological perspectives. The theory is based on a lot of empirical work together with research and consists of a system of concepts, which has showed its stability over a span of time.

3.1.1 Preparing

When I prepared C-04, I planned how to organise the activities which should lead to the theoretical concepts. My course plan called for quite a few different elements to develop together. The format had to follow the guiding principle, and the content should emphasise how mathematics communication was used in the work with the open tasks.

The goal with the workshop was to offer the 'teachers' the two theoretical concepts: The epistemological triangle and the interaction flowchart. These concepts were meant as effective analytical tools for the teachers in their work with observation, interpretation and understanding of their pupils' work and utterances, when the pupils express their mathematics knowledge. I imagined that these theoretical concepts were quite difficult for the teachers to use for observation, but I hoped that the activities they just took part in, and their opportunity to ask questions during the presentation, would make the models accessible to them.

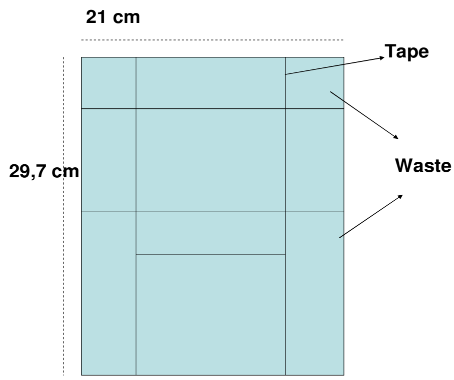
I organised work that the teachers could do alone, in small groups or in plenary sessions. My belief was/is that before the individual can supplement the group work, they should be allowed to work alone and establish their own understanding and opinion; a method the teachers should be able to practice back in their own classes. This would ensure that each participant brought an understanding to the group and this 'understanding' should be the objects of the group's work.

For some teachers, it is beyond their normal limits to solve tasks at an in-service course. Perhaps the situation is too risky for someone who is afraid to reveal possible weaknesses in their mathematical skills. To take this anxiety into account, I organised the problem-solving as a kind of role-play. The 'teachers' did not have to pretend to be somebody else and they should still do their best, but they should act like pupils who were asked to solve a mathematics task. In this way, the situation became like a school class situation, which was in accordance with my plans.

I aimed to design situations, where the teachers experienced 'what it was like to be a pupil' with the same requirements that their pupils had, and in such a way that their own habits became apparent to them. This role-play was later used in the reflection process. We could all take part in the simulated situations, and reactions could be analysed afterwards. This meant that we did not work with situations from the teachers' classes, but only with observations from simulated situations at this particular in-service course. In some of the situations, different teachers should solve tasks, while others were observing with different points in focus. The methods were all based on the guiding principles.

For the open task, I chose to use the task with the chocolate box – which I used before – to demonstrate the principle for the IF and ET.

The chocolate box
The owner of a factory wishes to produce a box for chocolate. He wants a box, which can be produced from a single piece of cardboard that measures 21.0 cm x 29.7 cm. He wants this design:



He asks his consultants to design the box with the following requirements:

1. Waste as little cardboard as possible
2. Use tape for the corners, but as little as possible (Notice: Cardboard and tape are both very expensive)
3. The volume must be maximised

The problem is: How should the box be designed to satisfy the factory owner?

One sheet A4 paper should be used to make a box with the largest possible volume, the least waste and least height. The problem should be solved in a group within 40 minutes. The openness in this task makes is impossible to fulfil all three requirements at the same time; therefore the solvers have to prioritize the requirements they find more important than the others and eventually present different solutions and explanations. This could be viewed as a closed task with no solution, but even though many teachers understand that it is impossible to fulfil all the requirements, they always try to find a way to produce a box that fulfil at least some of the requirements. The task was meant to be a puzzle with the added problem that a complete solution was 'impossible', therefore the requirements were to decide and explain what to focus on.

The groups communicated about various strategies for how to solve the problem. My expectation of this group work was that the teachers were able to communicate about a strategy for how to model the problem; this means that they were able to see the problem immediately and from there discuss how to choose condition and to calculate what was necessary.

If the task was analysed with functional notation, three functions were needed, where x is the length of tape in one corner (the height), and therefore it only can be positive:

- The use of tape: $h = 4x$
- The waste of cardboard: $A = 2(x2 + 14.85x) = 2x2 + 29.7x$
- The volume: $V = x(21 – 2x)(14.85 – x) = 2x3 – 50.7x2 + 311.85x$

The results can be compared through functions and graphical drawings. To make it less abstract, one can make a model of the box out of paper, or the task could be solved with tables of numbers for each new value of the height and then compared. My expectation of how the 'teachers' would use mathematics to solve the task ranged from just making a box, without much explanation, to a study and analysis of the functions, including description of increasing and decreasing intervals, and computation of derivatives, which again could be a basis for different solutions. The objective is not just to determine whether the teachers are able to solve the task or not, but also to find out, how the observing teachers are able to follow such a process and describe it. The focus is on the discourse, how the 'teachers' communicate when they solve the task and how they discuss this process afterwards.

Inspired by the way Niss describes aspects of the Danish KOM project (Niss, 2003), we can inspect what the teachers have to do, in order to solve the chocolate box problem:

- They should be able to perform mathematical modelling in the context of the 'practical' situation with the limitations given, which means to ask questions and solve problems pertinent to the situation. What dimensions could the box have to fulfil the requirements, and why.
- They should be able to think mathematically to find the kinds of answers they need to solve the problems, here dealing with: different types of functions. Furthermore, whether it is at all possible to fulfil the three requirements at the same time, or what kinds of compromises are needed.
- They should be able to pose, specify and solve mathematical problems as open problems. This means that they should establish the functions and understand what role these functions play in solving the problem. It could be graphical or just as a solution to the equations.
- They should be able to reason mathematically and justify statements, solutions and conclusions. They should, in this case, convince each other about their choice of methods and why one result is better than another.
- They should be able to choose different mathematical representations and to 'translate' between them. The box could be made as a physical model, or the functions could be shown graphically or just in tables. In each case, the teachers should be able to 'translate' between them.
- They should be able to handle mathematical symbolism and formalism when they draw the graphs or make the schemes.
- They should be able to communicate the key issues for each solutions and why one proposal or argument is better than another one, including have the ability to listen and to understand the others' argumentation.
- They should be able to use mathematical tools such as a calculator or computer program, and be aware of their aims.

The original setup for the task called for one observer. I changed this to three observers who were assigned different things to look for. The first observer (Ob1) was only asked to use skills for the observation that he already used in his daily teaching in his own classes. Ob1's task was: Observe how the group works and note what you find relevant to be aware of. The second observer (Ob2) was asked to look for the mathematics used in the solving process; this meant to determine what mathematical expressions were used, combined with what signs or representations were attached to the utterances, and in which concept categories the signs and calculations belonged (the ET). The third observer (Ob3) was asked to look for the discourse interaction in the group (IF); he should take note of who spoke with whom and how, and how it influenced the space for mathematical inspiration among the solvers. For each observer, I created an instruction scheme to inform them about what to do and how. I did not use the words Interaction Flowchart or Epistemological triangle in these instructions; in fact in the scheme and the text I simplified what to do.

'Kanal' / Typen af udtalelse	Privat	Interpersonel
Re-aktiv	↑	↖ ↗
Pro-aktiv	↓	↙ ↘

→ 'Konkret' tale   - - - -► Meta niveau   → Uklart

Figure 30: The Danish table used during C-04 for Interaction Flowchart

The table relied heavily on the original IF's descriptions. The observers had to distinguish between three different utterances through using the different arrow types: 1. object-level re- or pro-active 2. meta-level re- or pro-active 3. unclear nature. They should also determine whether the utterance was private or interpersonal. In addition to the table, there was an explanation of what it meant to be re-active or pro-active, taken from the original text.

The table for the EP was not set up as a triangle, but in columns so that it was made easy to write down the observations.

Concept	Reference context	Signs/Symbols/Representation

Figure 31: The Danish table used during C-04 for the Epistemological Triangle

In addition I showed them the following example:

Functions	Equation of first degree	Graphical sign

I wrote that they should look for coherence among these three terms. Along with the tables, I prepared to give the observers a brief instruction, aimed of making them comfortable with their task.

The meta-didactical transposition for both the concepts IF and ET was that I transformed a research idea written for a research paper into teachable activities carried out on an in-service course in such a way that these activities could be used in school classes. The IF was original presented, by Kieran and Sfard, as a tool for researchers to use along with video- or audiotapes. I transposed it to a tool for the teachers by making a table that was supposed to help them in their observations. The intention with the ET table was to help the observers to note the words and the signs used in relation to mathematical concepts. The table simplified the ET as well in the way that each row was meant for one single observation situation. Normally the framework is to be aware of an epistemological process, which could not be 'caught' in one shot, but which the 'teachers' were informed about in the lecture about the ET.

The time schedule for this workshop included an instruction (5 minutes), solving the task (40 minutes), the observers and the groups discussed the results (10 minutes), the observers presented their observations in a plenary session (5-10 minutes for each group), the teacher educator (me) presented the theoretical framework of the ET and IF (20-30 minutes), and finally a summary in plenum with a meta-discussion about what they got out of this assignment.

Such a workshop takes almost two hours, and the teachers are exhausted when it is over.

I followed the guiding principle in the planning process, as I organised activities based on the IF and ET. The goal was to hone the teachers' awareness of the different way discourse or communication can be observed; and to compare this with how the teacher normally would observe their pupils in their own classes. The methods were applicable to other teaching. The prepared activities would hopefully reveal habits and tacit experiences, and create a feedback mechanism for reflection via the three observers. The activities were scheduled to take place before the theoretical concepts were presented to the teachers.

3.1.2 Practice

The redesigned workshop with IF and EP was tried out for the first time on C-04, video taped on C-05, and run again on C-06. The following is a description of my interpretation of the practice based on logs, materials from the courses, tape recording and videotapes. The guiding principles were adhered to, and the chocolate box task was used in the form with three different observers each time. Details were changed for each new practice to improve the design.

C-04: At this course, the teachers followed my plans in a positive way. The groups were set up easily and work began. The groups appointed their own observers. These observers got a quick explanation and a manual for each category of observation. They did not know about the other observers' focus; they just had their own checklist to work from. During the workshop I realised what the observers found difficult – maybe too difficult - and what seemed to work well and why. I noticed that the tables for IF and ET were too difficult to fill in: the teachers did not use them, but made notes instead; therefore I decided to change them. I noted in my log:

> *The manuals were not a help to the teachers. They were all too difficult to use. The teachers were not able to use the different arrows; maybe they were not able to distinguish between the utterances at all. I need to change the table, so they only have to work with one type of arrows.*

The Danish ET table used the same terms as used in the original epistemological triangle, but the terms for the different categories seemed too complex to use for the observers, who somehow made relevant observation anyway. They more or less ignored the table and made their own signs to remember what happened in the event, and they used these notes in their presentation to the others. I decided that the table for the ET was OK, but that it was necessary that I expanded on my explanation. Ob1, who should use his own 'daily skills' was uncertain about what to look for; some of them did not know how to use 40 minutes looking at other 'teachers' solving a mathematics task. I interpret this as these observers were not used to observe their children for such a long time without saying anything, or they were maybe not used to listen to their pupils for so long. I observed another thing: the ob1 and ob3 made the same kind of observations. Ob1 made note of how the 'teachers' worked together much more than he noticed what kind of mathematics they used or how they used it. It seems as if the mathematics discourse was not a topic the teachers were used to look for, and I believe that they were more used to look for social behaviour in group work than to look for how the pupils worked and communicated with mathematics. I mentioned this observation immediately in the plenary session after the group presentations, and the 'teachers' recognised this observation and expressed that they normally did not have the time to observe in that way and that they were not aware of how they did it. Furthermore, I noticed how much the solution of the chocolate-box task differed from group to group. Some groups only made a box without any calculations, whereas other groups only made calculation and used a computer to present the results; other groups made both a box and calculations. The way the different groups solved this task was in a funny way analogous to the classes they taught; it turned out that the teachers who taught eighth and ninth grades used strategies from these grades, and teachers who normally taught lower grades also used the same strategies as they taught. The observing 'teachers' could be a little embarrassed to tell about the group's work, if there was very little mathematics to summarise. This was a difficult issue to discuss in plenum, because it apparently was embarrassing – my feeling – to exhibit low mathematical skills in front of colleagues.

Before the presentation of the theoretical concepts, we talked about the task, how it could be solved, what kind of open task it was, what was difficult etc. I asked if they would use such a task in the oral examination of their pupils. Most found it too difficult to use, expressions as 'when it was so difficult for us, how would the children do?' The openness of the task had maybe confused some of them, but none of them argued that it was impossible to fulfil all the requirements in the task. It was said by the 'teachers' that the significant task for them was to determine the different functions for volume, waste and the height. That was difficult for them, and took up their attention.

The presentation of the theoretical ideas cleared up what the 'teachers' tried to do during the observation. Their reactions were like these:

- Oh, that was what you meant
- Did I work with epistemology? What does that mean? Please spell it
- Now I see how I could use your table

I noticed that the tables caused more confusion than help, but it did not have a negative impact on their understanding of the theoretical part. On the contrary it seemed that their confusion made them more interested. I explained my intention with transposing these theoretical ideas into a teaching that could be useful for them as tools for observations of their own pupils. The discussion was lively and the teachers asked many questions concerning how to use the tools in their own teaching. They expressed worries about how they would be able to observe so many things when moving around among twenty-five pupils in the class, but they also said that it was an eye opener; especially that they realised that they looked more for social behaviour than to mathematical content in their daily teaching (this is a summary of my own log).

C-05: Before I tried this workshop out again in C-05, I changed the tables and found other ways to explain more clearly to the observers what their task would be. It was difficult to organise the groups, and to ensure that the observers understood their task. This time I made a table for the IF with only one kind of arrow to explain the utterances; I demonstrated how the observers should draw the arrow in different ways to show who the speaker talked to and whether it was pro- or re-active.

During C-05 I videotaped how a group of six teachers solved the chocolate box task. Three teachers solved the task (‘the ‘solvers’), while three other teachers observed them from different viewing positions. The three solvers were all mathematics teachers with more than twenty-five years experience in teaching mathematics in classes at all levels from grade 1 to10. The observers were younger teachers with one to seven years of experience; they were two men and one woman (I shall call them Claus, Torben and Mette). Claus was ob1, he was not specialised in mathematics, and had seven years of teaching experience; Mette was ob2 (ET), she was specialised as a mathematics teacher and had one and a half years of teaching experience. Torben was ob3 (IF), he was not specialised in mathematics, he had five years of experience as a teacher.

The three solvers sat at a large round table, which means that they were sitting a little like in a curved row; this placing meant that the person in the middle had access to both the other solvers, while they, in turns, had difficulties looking at each other’s papers and calculations. They were two men and one woman (I shall call them Anders, Bo and Ruth), Bo sat in the middle. The observers could walk around and look where they wanted, but were not allowed to talk during the 40 minutes the group had to solve the task. They were given tables and explanation of how to write down their observations.

Observation from a group work during C-05: After reading the initial stimulus material, the solvers discussed how to start the solution process. They decided to work with the waste of the cardboard, and the impact it had on the form of the chocolate box, and they discussed how this could be determined. They did not talk or agree about strategies for how to solve the task. Nor did they communicate about how to model the problem; only the part of how to determine the amount of the waste was in focus of their conversation. The following transcription shows the communication about which strategy to use to determine this waste. They seem to have a little power struggle about strategies. The episode contains utterances which show preoccupation with their own thoughts rather than with the exchange with their partners. In the beginning they discuss how to calculate the waste. It is difficult to understand exactly what they talk about because there are many unfinished sentences and they point at each other’s papers instead of saying what they do. In that way, I see it as a typical mathematical communication in the phase where the partners still need individual time to understand how they would solve it alone.

In the transcription (my translation):
‘…’ means a little pause or unfinished sentence,
(…) means that we jump in time,
(?) means unintelligible.

[11] ANDERS: Then there are 2cm^2 up here, then we normally know that… no, this drawing is totally wrong, it is 19, isn’t it?

[12] BO: No…

[13] ANDERS: What kind of nonsense is this, the breadth of our box - when it was 1 – was 27.7 divided by 2… it was that before…. Are you with me?

[14] RUTH: Yes, yes…

[15] ANDERS: It is then 13.85, it means that we get … oh, yeah, but then we could maybe take…

[16] BO: Two times 13…

[17] ANDERS: It is then 13.85 the two of them here… 13.85, then you should add the 2 cm and get 15.85…

[18] RUTH: Why are you adding the 2 cm?

[19] ANDERS: To take the stripe here – we need to make a system we could use… get it to fit in here.

[20] RUTH: But I don't understand why you add the 2…

[21] BO: But we have the system…

Anders talked about the waste in the corner if the height is 1 cm, and for two corners he got $2cm^2$. In [13] Anders explained how they found the breadth of the box when the height was 1 cm, but when he wanted to add 2 cm he was not able to explain his way of thinking. Bo wanted to multiply – he was maybe thinking of the symmetry in the box. They used pointing and gestures in their communication combined with their written calculations. They started out on their own and talked a little about their way of calculating. Anders had the leading role in the communication, until Ruth asked the questions about 'adding the 2' [18], which she never had an answer to. If I use the IF interpretation on this communication, we see the communication from another point of view:

ANDERS	BO	RUTH	
[11]			ANDERS: Than there are $2cm^2$ up here, then we normal know that… no, this drawing is totally wrong, it is 19, isn't it?
	[12]		BO: No…
[13]			ANDERS: What kind of nonsense is this, the breadth of our box - when it was 1 - was 27.7 divided by 2… it was that before… Are you with me?
		[14]	RUTH: Yes, yes…
[15]			ANDERS: It is then 13.85, it means that we get … oh, yeah, but then we could maybe take…
	[16]		BO: Two times 13…
[17]			ANDERS: It is then 13.85 the two of them here… 13.85, then you should add the 2 cm and get 15.85…
		[18]	RUTH: Why are you adding the 2 cm?
[19]			ANDERS: To take the stripe here – we need to make a system we could use… get it to fit in here
		[20]	RUTH: But I don't understand why you add the 2…
	[21]		BO: But we have the system…

Most of the communication is on the object-level. If we look at Anders' communication, we see that it is a mix of loud inner speaks and proactive utterances like 'are you with me?' [13]

which could also be just an empty polite phrase. Still, Anders is the one who took the initiative in the communication, and he led the calculation until he made a mistake. If we look at the arrows coming from him, we see that they are pro-active. If we look at Bo's arrows he has only re-active comments and the same goes for Ruth. Anders worked 'alone' [11] [13] [15]; he thought out loud, and he did not listen when Bo said 'taking twice' [16], instead he continued the adding of the '2', which Ruth did not understand and hence asked him about [18] [20], where [20] is a question on the meta-level. Anders' mistake, which he could not explain, meant that he went off into his own calculations, while Bo and Ruth stopped listening to him and began to work on their own. It was obvious to see that Anders worked with his own calculation and was not ready to explain anything or to communicate very much; he was in a phase where he needed to find out how he himself understood the task. Still he was a part of the communication in that Bo and Ruth asked him questions. Even though they communicated they were no help to each other; Anders' calculation [19][21] was not clear to Ruth, and maybe it was not clear to himself either – and to me it looks as if he makes a mistake, which he could not explain or resolve. Anders continued his work, and Ruth continued to 'stop' him or to suggest changes to what he was doing. I chose to bring the following communication because I interpreted it as a significant pattern of the cooperation. First, it shows the phase where first Anders and later Ruth are preoccupied with their own thoughts rather than with exchange with the partners, and furthermore it shows their mathematical strategies and skills.

The next phase is an 'engagement phase', where they try to convince each other about their strategy:

[25] RUTH: You can likewise say…

[26] ANDERS: Wait a minute I need to do… multiply with the height multiply 2

[27] RUTH: But when you moreover say that 29.7 is the length…

[28] ANDERS: Yes, yes, you can do it all, but if you do like this multiplying with 2h… such a formula you can complete. Then we don't have to think a lot more.

Anders was still not ready to listen to any other suggestions or to talk about strategies. He shared his thoughts by speaking them aloud. Ruth gave up and started to work on her own. Bo turned his energy to Anders and they worked together and pursued Anders' idea about finding a formula. After a while Ruth turned her energy back to the two men. In the beginning she just looked at their work. After a while she started to comment:

[58] RUTH: Plus the 4 there, if you should…

They talk simultaneously about Ruth's suggestion

[62] ANDERS: It is the height of the box…

[63] RUTH: Yes, it is the length of the box here…

[64] ANDERS: It is surely this number (point at his paper)

[65] RUTH: No, it is twice…

They discuss further

[67] RUTH: It will always be the length of the paper; it means it will always be 29.7 – 2h (she laughs)

[68] ANDERS: (turns away from Ruth and Bo) It might be the case…

Ruth and Bo continued talking about the new results. After a while Anders leaned forward again to take part of what happened between Bo and Ruth and to look at their sheet of paper, which they discussed. They all took part in the discussion now, but Anders was not convinced of Ruth's solution:

[80] ANDERS: No, then we forget that we need to multiply with 2 up here, that's why the 2 has disappeared.

[81] RUTH: Not if we look after the two of them (pointed at her paper) … yes, it is correct

Anders leaned back again and worked on his own sheet. After a while he turned back to the group.

[84] ANDERS: This formula isn't simpler then what we had before, the opposite…

[85] BO: This here is more general

[86] ANDERS: No, the one we had before was (?) at once…

[87] RUTH: How did you find the part, the counterpart?

[88] BO: We know the waste

[89] RUTH: Yes

[90] BO: It was the height, I guess

[91] RUTH: Yes… yes, if you want to find a general formula, you need to…

[92] BO: Yes, but let us see if we can manage it… it is… let me see 4 + 27.7 and it is 31.7, isn't it (?) Isn't it correct that the waste is 31.7? (to Anders) what are you saying?

[93] ANDERS: I am trying to calculate a little (?) so I can say… it gives…

[94] BO: You are allowed to do that… we are all sitting with our own thoughts alone; can't we have a little more cooperation? (turned his head to both Anders and Ruth)

[95] RUTH: I try this… or else I can't…

They continued their separate work.

The action and the communication from [58] to [68] is the beginning of the discussion between Anders and Ruth about how to find the right results of the waste in an easy way. Ruth worked with her way alone, and when she turned to the group again, she listened in the beginning, but after a while she tried to convince the other two about her strategy [67] about how to find the 'length of the waste'. It seems that it started a kind of power struggle, when Anders [84] expressed his dissatisfaction with Ruth's formula. The different strategies involved for Ruth's solution subtracting '2 x h' from the length of the cardboard in contrast to finding all the waste calculating 2 x (half the length) + 4 (Bo and Anders') – The calculations were still only for the height to be 1 cm: [92] Bo managed to find the waste with the height 1, and his strategy was to add the four corners with 2 x (half the papers length).

If we analyse the part from [80] to [95] with the IF tools, we can see some of their 'fights' and way of working, separately and in cooperation.

ANDERS	BO	RUTH	
[80]			ANDERS: No, then we forget that we need to multiply with 2 up here, that's why the 2 has disappeared.
		[81]	RUTH: Not if we look after the two of them (point at her paper) … yes, it is correct
[84]			ANDERS: This formula isn't simpler then what we had before, the opposite…
	[85]		BO: This here is more general
[86]			ANDERS: No, the one we had before was (?) at once…
		[87]	RUTH: How did you find the part, the counterpart?
	[88]		BO: We know the waste
		[89]	RUTH: Yes
	[90]		BO: It was the height, I guess
		[91]	RUTH: Yes… yes, if you want to find a general formula, you need to…
	[92]		BO: Yes, but let us see if we can manage it… it is… let me see 4 + 27.7 and it is 31.7, isn't it (?) Isn't it correct that the waste is 31.7?
	[92]		BO: what are you saying?
[93]			ANDERS: I am trying to calculate a little (?) so I can say… it gives…
	[94]		BO: You are allowed to do that… we are all sitting with our own thoughts alone; can't we have a little more cooperation?
		[95]	RUTH: I try this… or else I can't…

The arrows coming from Anders show now that his utterances were mostly re-active. His role in the group has changed. He started to criticise Ruth's strategy [84] and expressed more satisfaction with the one he had before. But now Ruth and Bo were in agreement and seemed to share an understanding. More of the arrows coming from Bo are pro-active, both related to mathematics and the social collaboration [94]. The arrows coming from Ruth show that she is both re-active and pro-active. She defended her strategy and she tried to explain it. The content is mostly power struggling about very similar strategies. The following communication was chosen because it demonstrates their finding of the function for the waste, how they understand it, and how they communicated about that understanding.

After some individual work, they communicated again about the equation for the waste area:

[115] RUTH: Then we have the formula

[116] ANDERS: What does it say? What did you write, there? (points at Ruth's paper)

[117] BO: 2h2 + 29.7h

[118] ANDERS: Plus 29.7h (write simultaneously)

[119] BO: It is the waste…

[120] RUTH: It means that…

[121] BO: And when will it be largest?

[122] RUTH: Now, we can say that it was the two of them we have here…

The discussion now concerned what to do; Ruth was goal oriented and wanted to find out what the waste became when the volume was biggest, while Bo and Anders wanted to see what happened with the volume when the height increased over a longer interval than just to find out where it was biggest or smallest. Ruth worked on her own and made a box of paper, while Bo and Anders communicated above their calculations. After a while Ruth participated again.

[150] BO: 160, the waste grows more and more

[151] RUTH: Yes, the more height the more waste

[152] BO: Yes, the equation says that. Is it right?

[153] RUTH: Yes, it seems to me that we can see it. Look at this one, I made…

[154] BO: We were there, where the volume was biggest

[155] RUTH: Yes, what do you think of, what was it? (looks at Bo's paper) It was a fiver.

[156] BO: There I have 123 plus… it is 275… what about the sixes, what is the result? What does our formula says? If the formula is correct.

[157 RUTH: yes, but it will… the more you take… (takes her box and shows how it will change)

[158] BO: Until it is such a high little affair (show with his hand a high small thing) but so high (moved his hand as high as possible)

Until now it seemed as Bo was not convinced of the formula [152], but with his own gesture it seemed as he understood. But later on he said:

[161] BO: Thus, the more height we want in our…

[162] RUTH: The more waste…

[163] BO: I don't understand at all…

[164] RUTH: But you have to…

(…)

[167] BO: In the end there isn't any chocolate that can fit the box (looks at Anders)

[168] ANDERS: I can't… you are so fast when calculating

They continued the work, where Ruth started to concentrate on the tape. She finished the box make of an A4 sheet of paper and calculated how much tape she needed; she did not look at the original model, but suggested different solutions where to put the tape. Both Anders and Bo did not care; they were more occupied calculating new volumes and areas of the waste. They found some mistakes they did before and agreed that it went a little too fast. Anders expressed at the end that he did not like the task and that he was irritated from the beginning:

[246] ANDERS: I have to make such a stupid box, I can't be bothered.

Both Ruth and Bo do not understand and point out that they were all very engaged in the calculation and process, but the collaboration was not as good as expected. The time is up. The observers try to interrupt with some comments; it is difficult because the calculation group wants to reflect among themselves.

This communication shows how Anders, Bo and Ruth collaborated and communicated; first how Bo and Ruth communicated and understood each other [115] to [122] and later on when Bo changed to work with Anders. They both wanted to investigate what happened with the volume for several values of the height and waste, while Ruth only wanted to find out what the height was for the largest volume. From [150] to [158] Ruth was back and succeeded in winning back Bo's attention. He seemed to acquire an understanding of what happened when the height was increasing. Again Anders was not a part of the communication and he was not able to follow the way Ruth and Bo calculated. Bo was in the middle, and switched his attention between Anders and Ruth, who fought about whose strategy to use. Their strategies for solving the task seemed very similar to what could be found in an eighth, ninth or tenth grade, where these teachers normally taught. They had the same difficulties and discussions about finding the waste as a function of the height as some pupils would have; they wanted to use the calculation for the height being 1 cm for finding a general formula, and this gave them trouble. Ruth found the formula, but Bo had trouble understanding it, and Anders never understood what was going on in Ruth's calculations, their collaboration was characterised by their own understanding more than by a will to try to understand the others. Bo asked for collaboration [92], but Anders and Ruth were not ready, and none of them, it seemed, knew how to do it. This collaboration situation was similar to pupils' collaboration in mathematics group work. Now, it is maybe understandable for pupils that they have trouble helping each other and with how to listen and collaborate, but here we have teachers, who require of their pupils that they should be able to communicate and collaborate to find a solution for mathematics tasks; this is a part of the curriculum requirements for the oral maths exam. My point is that this is another similarity between the two kinds of classrooms (schools and in-service). Not only are they similar in form and 'behaviour' but also in the content of mathematical skills. These findings show what many teacher educators know: that the 'teachers' on in-service training having similar skills in mathematics as do the pupils they teach. We learn when we teach, and therefore the teachers learn the mathematics used in the textbook in the actual grades in which they teach, and that is exactly the mathematics they use when they solve the task on the course. Even if they learned more mathematics at pre-service education, they do not use it here; maybe it is lost maybe not, but it is not triggered to come to work in this situation. Another explanation could be that they never really learned to understand mathematics; they only learned to solve maths tasks, and that is what they continue to do. They do not know to do it in different ways.

If the teachers become aware of such findings or interpretations, they could hopefully become aware of the difficulties in group work. We have to pay attention to both the form and the content. Power-struggles are not unusual among pupils when they are engaged, but communication during collaboration is just expected to work, while on closer inspection, it rarely does. Communicating about mathematics and solving the task at the same time is difficult. It has to be learned and therefore to be taught. The individual needs to identify the problems alone and to figure out when to talk and inspire each other, and when and how to talk and listen. It is obvious that the solvers' work does not correspond with effective cooperation. This experience with their own knowledge could be a first step to motivate the 'teachers' to work on to improving such skills, or it could be the opposite should this be too difficult for the 'teacher' to learn. Yet, the 'teachers' are unaware that they do not know it, and that they solved the problem in the same way their students do. They do not have the meta-knowledge about their own habits or skills, maybe because they were never forced to reflect on it, or because they never saw it as important.

In the last minute, Anders said that he did not like the task; I believe that he did not like the process. He did not like Ruth's strategy, maybe because he was not ready to listen or to explain his own thoughts, when the others wanted him to collaborate. He was in the middle of his own process of solving 'his task', which took some time and energy for him, maybe more than he expected. When he was not given the time he needed to understand the problem, he felt ill at ease and ascribed it to the 'stupid task'. Unfortunately, I did not know all that when we had the plenary discussion; I noticed it later when I watched the videotape. As for the details about the mathematics content, I will analyse it after we 'listen' to comments from the observers and before the final conclusion and hypothesis, which will come after the next analysis.

Observers' comments on C-05: I selected the next part to demonstrate how the observers explained their interpretation to the group immediately after the group work had finished. The interpretation of this communication will come after the description.

Claus was ob1 and observed from his own point of view; Mette was ob2, who observed the mathematical utterances, while Torben was ob3, and looked at the communication interaction.

[264] CLAUS: I want to focus on the communication and your attitude Anders. Ruth tried several times to participate, but each time you interrupted her. You cut her off before she could explain her ideas (Anders nods with approval) and it is important that you take your time to listen to what she has to say. I know from myself…
[265] ANDERS: Yes, I know…
[266] CLAUS: Yes, you become so preoccupied by what you want to say that you don't listen…
[267] ANDERS: Yes, I know this is a weakness I have
[268] CLAUS: If you want to benefit from your work, you have to practice to become a better listener

They began the reflection and discussed when it was possible to listen and when not. Anders admitted that he was not able to listen when he did calculation. He was in his own world and not ready to listen to others' way of thinking. It was very understandable, and this was the reason why I encouraged them to use time on their own, before they started the collaboration, but the group did not follow that strategy.
[275] ANDERS: But I can see that it offends others. I know it.

Claus explained how he saw the cooperation, that Bo had a mediators position and that he was the one who listened to Ruth, who brought the ideas. Ruth felt the need to explain
[285] RUTH: I think it is my nature to think like that, I need to have it in my hands. I remember when you cut me off, and then I decided to just make the box.

They discussed how the cooperation felt like. Anders expressed that it went all too fast, he was unable to follow Bo and Ruth, and then he decided not to care about it. Claus asked Anders:
[290] CLAUS: Would you be able to solve the task or do anything else to come into the cooperation instead of just working alone as you did, could you think of alternatives to what you did?

[291] ANDERS: Yes, I could have asked him to stop and explain to me what he did. I could have asked quietly.
[292] METTE: You say in fact now that it went too fast…

Now the whole group discusses what happened more or less simultaneously.
[312] METTE: (…) why doesn't Ruth just say: Now I want to say something (she bangs the table) you are interrupted a million times, where you just say 'eerrrm' or 'oops'. I wondered what you wanted to say several times.
(…)
[320] METTE: Because Anders is difficult and Bo is difficult too. You have two difficult men to deal with when you…

Even though Mette had the task to look for 'mathematics', she had to tell what she felt about the interaction between the 'teachers' working. They continued to talk of what happened and why for several minutes.
[343] METTE: The meaning with this task, then, is that you are yourself and get some critical feedback, which means that you will think differently in the future.
[344] ANDERS: Yes, but…
[345] METTE: I was very impressed and hoped that I will be so capable, when I have been a teacher for many years, really; I thought of this, and I want to become capable as well; to work so fast… it could be the case that you thought it took a long time with that formula, but I am afraid I would still be working on it.

Mette expressed how difficult it was for her to follow the calculations in the group. She questioned if she would be able to do it herself, if she had the time to think. I wonder how that could be a question. Mette is specialised as a mathematics teacher and this task did not require any particular skills that could not be expected from a pupil in eighth, ninth or tenth grade. Maybe she was not trained to observe how pupils work and how to listen, or maybe she did not use time to find out what the task concerned, or she did not have any strategy to solve such a task; she has only little (1½ years) teaching experience (according to the hypothesis that the teachers learn when and what they teach), or she was polite because she was a little rough to Anders, but she repeated her impression several times. It is difficult to stipulate why Mette reacted in that way. The reaction to her comments in the big group (including observers) was that she had other skills, and it would have been difficult for them as well to look for the mathematics. Their conclusion was that younger teachers were not as trained in mathematics as teachers were earlier, but younger teachers maybe had other skills. What those skills could be they did not discuss. The consequence was that the group did not discuss the mathematics used during the task solving, that it was an open task, whether it was a good task or what kind of mathematics was necessary; they concentrated on the social behaviour in the group.

It is apparently more difficult to follow others' way of thinking than to think and solve problems on your own. In this group, especially, it was difficult to follow their thoughts and ideas because their communication was so full of unfinished sentences and speaking loud inner-thoughts. Mette continued to praise the solvers' mathematical capacity, and her criticism only concerned their cooperation, not their mathematical strategies or their's way of communicating about them. She had to look for the mathematics in relation to the epistemological triangle (ET), but she never connected their expressions about the objects with the signs. If I fill in some of their expressions in the ET, we get some different triangles:

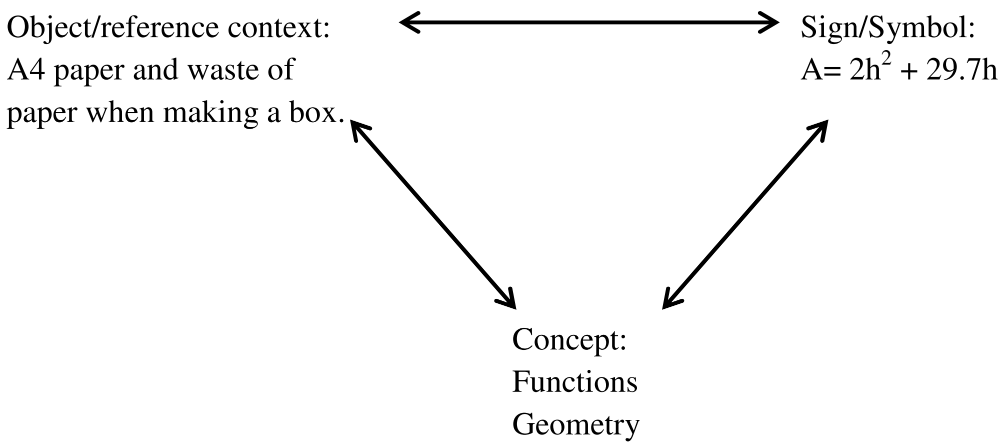


Figure 32: The epistemological triangle of the work with the box

The decision for what kind of concepts were in play is an interpretation of what the group worked with. Their communication and calculation focused on of how to find the waste, which had a geometric perspective, while finding a general formula for the waste concerned the concept of 'functions'. The teachers were not very precise in their communication and worked out a function or formula for the area of the waste ending with A = 2h2 + 29.7h, where 'A' is the waste and h is the height; they made a table to find the change of the area; they did not use any graphical representations.

Another epistemological triangle could look like this:

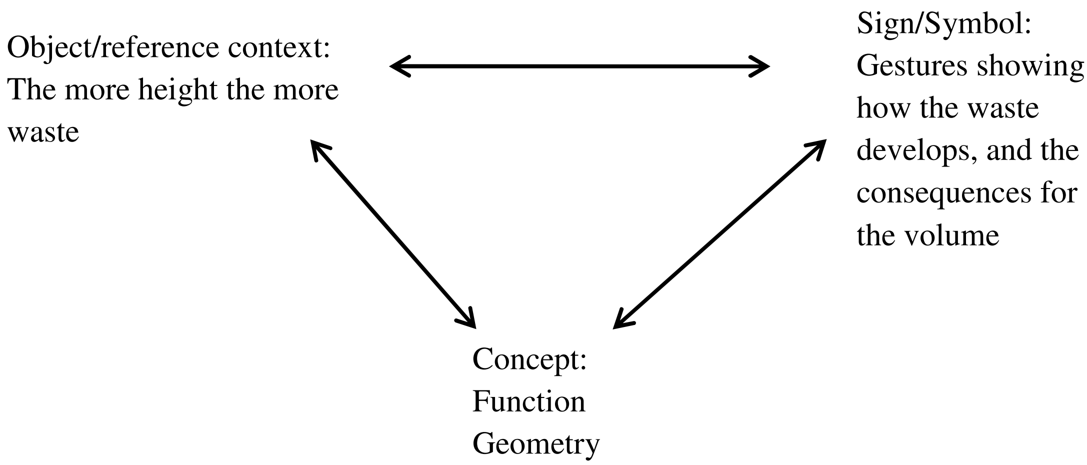


Figure 33: The epistemological triangle of the work with the box

They managed to find the equation for a function of the variable waste and to make a table. How the waste and height were connected was understood by Ruth and maybe by Bo, interpreted from his gesture. It took them 40 minutes to obtain that result. Ruth needed the real box in her hands to understand how it changed with different heights. Comparing the results from this group with how other groups solved the task, we can notice differences which could be an expression of the 'teachers' mathematical level. If we observe a group of pupils over a period of time, we can look for an epistemological improvement in the pupil's expression, how the concept is described in words and signs and how new knowledge grow out of existing knowledge. The teachers can be aware of the kind of expressions they expect in mathematics communication and on the course discuss these expectations and the values connected to them. This episode above shows how the utterances combined concepts and signs, but not what the development would look like. Still, this task made it possible to compare with other groups, which I did in the lecture about the ET. The in-service course was about the oral exam and about evaluation of how the pupils communicated about mathematics. The use of ET gave the 'teachers' a tool, which could help them find out what kind of words and representations were related to a certain concept.

In this group work it was difficult for the observers to follows what kind of mathematics the solvers used and why. If this is the case, it is even more difficult to evaluate what kind of development the teachers can expect from their pupils' acquisition of mathematics. Clearly, to build the box out of paper was different from just having it as a drawing, and we can see in Ruth's comments what this meant to her work. Ruth made the box and delivered the formula to calculate the area of the waste, and said that it was easier for her to find the answer because she had the box in her hands. During the task, the solvers made many small mistakes, which could easily confuse an untrained observer. It was the first time Mette was to look for mathematics when other persons worked, she said. This 'confession' tell us a little about her practice. She did not have an analytical tool to use for observing any mathematical work. The solvers were not very precise in their explanations, they pointed to the paper instead of using words, and they had difficulties explaining their strategies to each other. This is no difference from a group of pupils discussing and pointing when they solve a mathematical task. The 'teachers' had different strategies and ways to solve the problems, but it seemed that they lacked the skills to make a working plan for any modelling or to use each other's competencies in a meaningful way. Mette was unable to see what happened from a mathematical perspective. We may expect that Mette would have solved the problem before a school lesson and knew how it could be solved, but she would still have trouble if some of her pupils wanted to solve it in a different matter.

Both IF and ET were developed in school classes, and the discussion among the solvers could likely take place in a school class as well. Therefore the analytical tools can be used on two levels of in-service education: For use in an observation task and as a tool to be taught for later use. In the group, it became obvious that it was difficult to communicate about mathematical strategies. The solvers felt how difficult it was to work in a group and discussed their own difficulties, the observers had difficulties as well, but in this discussion phase none of them discussed how to resolve their difficulties or improve their skills; they did not talk about alternatives. What the solvers learned from the observers was that they were imprecise and did not listen to each other. When somebody is preoccupied with a task, it is difficult to speak. Their gestures played a role for their understanding as well, e.g. when Bo 'gestured a box' with his hands and said [158] "Until it is such a high affair but so high". As for the cooperation, it seemed that they wanted to understand the problems before sharing their thoughts, which is maybe a problem some pupils have as well: To have enough time to understand before they are required to give an explanation. But the group did not talk about that.

The solvers explained their behaviour more than why they reasoned and calculated as they did, when the big group discussed and reflected. Ruth explained why she made the box, but in the group discussion nobody reflected upon the strategies they used or how it could be done otherwise. They tried to explain what was going on from a social perspective. Because of Mette's insufficient skills, they generalised that the younger generation of mathematics teachers did not have the competencies to calculate so well and concluded that they perhaps had other useful skills. They never said what kind of skills or competencies that could be, or why it should be like that. In Mette's presentation in the plenary session, she said that the solving group talked about volume, equations, formulae, squares, cubic, functions, power, reduction and parentheses, but she did not mention or show any signs to fill in an ET; she mentioned again that she was impressed by their work. Ob3, who looked at the IF, did not say much in the group discussion, but in the following plenary presentation, he explained and demonstrated with some new models. He drew on the white board how he saw that the group consisted of three different people with different approaches to the task. His drawing consisted of three circles; the two of them he called the number crunchers and the third was the practical designer – here he meant Ruth. All three of them were preoccupied with solving the task, but had different objectives. Ruth wanted to make the box according to the rules framed in the task, while the two men were more interested in the mathematics that arose when they started to solve the task, he said. He explained how they disappeared into that topic and left Ruth outside with her solutions and questions. He did not use the IF table, but when he explained his observations he used another model. His models led to a different result than the IF, but maybe not so different after all: that the solvers engaged in a power struggle and were unable to work together as, they had different objectives. It turned out that the IF arrows were still difficult to understand and use, even though I tried to make an easier version. When I worked of the table to fill it out, looking at the videos, I understand why it was so difficult to do it on the fly, but my argument at the time was according to my GP that I did not want to explain too much in the beginning; therefore I only made a short oral explanation about it and a little more on the paper.

During C-05, where the video was made, I tried to simplify the tables from C-04. The reports and summaries from the other groups showed very much the same: the tables, I made for ET and IF were too difficult to use. Still, the observers understood more or less what to look for, we heard in their presentations. In the summaries from the groups, the observers expressed that just being an observer was an eye opener in the way that they gained new insights about how their colleagues worked, and that they recognised that they had similar habits. In this phase the teachers who solved the task did not say much.

Solving the task in common was done after all the presentations. I showed the 'teachers' a sheet about how the three functions could look in tables and in graphical representation to discuss how a solution could look. There were not many questions during this explanation. Mette said that she was impressed in the plenary presentation, but when I showed a solution, she did not find it difficult. The openness made it difficult to find out what solutions to look for, they said, and the height was ignored by several groups.

The presentation of the theoretical ideas was the final step. When I presented the instruction about the IF, the observers saw how their own models fitted into the original idea. It was obvious that the observers asked the most questions. The solvers were the pupils, who were observed and maybe felt compromised. The IF system, they said, served as an analytical tool for what they felt they already did, while the ET was quite new to them. How to communicate about mathematics was normally not on the agenda when they observed the pupils, they said, and not when they observed the 'teachers' either. Asked why, they came up with different explanations. One said that he was only interested in the results, another that he did not think about it, or that it was important to educate the pupils, and that this took a lot of their concentration; maybe it was not necessary to educate the pupils so much if they were preoccupied with mathematics. When we met again on C-05, Mette told us all how she used an observation model in her eighth grades: She let pupils observe other pupils discussing a mathematics task, and had the observers focus on mathematical expression and signs. If she had difficulties, she let her pupils try the same. When asked how it went, she answered that she was satisfied, and that the pupils did a good job, which led to interesting discussions.

Interpretation of C-05: Analysis of what happened at C-05 leads to a number of questions:
- How did the guiding principles work?
- What kind of tacit knowledge became apparent for the teachers and for me?
- How should I work with the redesign for the next cycle?

The questions vary depending on who asks them, the researcher or the teacher educator. As a researcher, I want to find out how a meta-didactical transposition can be carried through in an effective way and how 'teachers' responded to the content. The guiding principles worked as effective scaffolding. The activities made several 'weaknesses' or habits on different levels apparent to the 'teachers': Both regarding the mathematics and how to observe the kinds of mathematics and communication they wanted the pupils to engage in. I saw these details more clearly, because I videotaped the situation. As for the order, it was problematic that the observers did not possess the right skills for observing, but it was a learning process for them, and at the same time a weakness to the observation of the group.

As a teacher educator, I had to find out what was realistic to expect from such a course, and what I wanted to emphasise. I was surprised that the mathematics seemed so difficult for the 'teachers' and the observers we followed. I have noticed this before when using this task, but I never videotaped it before to find out how what caused the difficulties. As a researcher, I see that the teachers use the same strategies as the pupils they teach and have experiences with. Their daily teaching has made them good at working on that level, and they did not feel any need to use more sophisticated mathematics for this task. School mathematics is dominated by tasks and this 'tasks discourse' (Mellin-Olsen, 1996) influences not only the skills the pupils develop but also the teachers' skills in solving mathematical problems; they are good at the textbook they teach. Working in collaboration was also difficult in the same way as Sfard and Kieran describes it in their article (Sfard and Kieran, 2001): The individual pupil is more preoccupied by his own task than with helping others; cooperative work has to be learned, but the teachers need to know how and what the problem is; IF could be a tool for help.

I was surprised that Mette said that she was impressed with the mathematics that the solvers used. I could not bring it up at C-05, because I did not know this when we summarised our observations (I saw it when watching the video), but I did use it on C-06. Later on I have met several young teachers in other in-service courses and asked them how they felt about their mathematical skills. Their answers have generally been that what they learned at the pre-service training was not as useful as they could wish, and they had to relearn what kind of mathematics was useful in the classes they had to teach. For example one mentioned the names of different geometric angles, while someone else mentioned the many ways to find percentages. They said that being a teacher required other mathematics skills than they had learned during pre-service education; one said that she participated in solving the final national writing tasks for elementary school, which was difficult for her, but it was necessary that she was able to solve them before she could teach lower secondary classes. This pattern, that school mathematics requires other skills than the students learn at pre-service education, may explain some of Mette's difficulties. She was in a phase between what she (maybe) learned during teacher training and what she had not yet learned from teaching. These findings are surprising; not only do the pupils learn that mathematics is just to solve task after task, but the teachers are victims in this game as well, if they can only master the maths they teach themselves.

Another thing I found surprising was that the observers, who should use the 'normal' way of observing pupils doing mathematics, only looked for the interaction between the solvers. I registered this already on C-04, and this was repeated in all later groups.

As for teaching the format, I needed to change the tables. It was a weakness of the course that the IF system was so difficult to use, even though the teachers found other ways to observe the interactions. When I analysed this by looking at the videotapes, I could see the difference in doing the observation in the real situation and looking at it on a videotape, where it was possible to go back and see again what really happened. I could use the tools and the IF-arrows, because I had the videotape, so I could see and transcribe what was said and how. The teachers had the difficult task to do this on the fly. We expect and require that the teachers master the complexity in teaching, however to 'survive' as a teachers, they must develop habits and routines, but not all of these habits are functional when teaching mathematics – and Danish maths teachers do not use videotapes to look at their own teaching or how the pupils solve mathematical problems.

My conclusions from the time between the two cycles of the in-service courses were split into my role as a teacher educator and my role as a researcher. My conclusion as a teacher educator was that studying the videotapes gave me vital knowledge to develop my next teaching cycle. I believe that the course taught the participants to focus on communication but not on mathematical communication, and the teachers were not taught new ways to communicate to a sufficient extent. I showed them the analytical tools IF and ET, which they found interesting, but it was difficult for them get to know how they would use them. I could use this insight as a teacher educator in a discussion on my next workshop. If the new groups at C-06 had any of these problems, I would discuss how they could resolve them. I decided to do the same task again, even thought it was difficult for the 'teachers' to solve. To focus difficulties made it more apparent to me to understand and find relevant teaching to answer these difficulties.

My conclusions as a researcher was that the guiding principles worked very well in many respects: The theoretical concepts were identified before the activities, the activities worked more or less as planned, several of the teachers' habits became apparent to both themselves and to me, but how to work with the habits in a reflected way was still a challenge. One weak point was that it was difficult to know what went on in the groups when several groups worked simultaneously. For the next course I could only use what I videotaped. At the course, I could only observe how the teachers worked when they worked, and what the observers told me at the summaries in the plenary sessions. That the activities had to be presented before the theoretical concepts made the tables difficult. This was a weakness, I could not resolve.

C-06: In my preparation and realisation of C-06, I changed the tables for the observers to be simpler, and I explained in greater detail what they should look for: Who spoke with whom and how, re-actively or pro-actively, and especially to take note when someone spoke to him or herself. I tried to make the tables for both IF and ET so easy that the teachers could use them on the fly. As for the IF, I made a table with just one type of arrow, and I demonstrated how to use it. The meta-didactical transposition of the original IF table has been more dramatic in this edition, to such an extent that the transformation is pretty unlike the original IF than it was in the earlier versions.

1	2	3
(2,3) ↓	**(1)** ↑	
		(2) →

Figure 34: Table for IF on C-06. Arrows for utterance: Vertical up is private re-active and down is private pro-active. Horizontal is interpersonal re-active or pro-active. The table shows that (1) speaks with a private re-active utterance to (2) and (3), (2) speaks privately re-actively to (1) and (3) speaks interactively to (2).

In fact the table helped the observer explain to the others in the group as well as in the plenary session, what ob3 (IF observation) saw, when she observed the group communication. The IF-schemes were filled in with names so it was obvious who was speaking and how. The problem was still that the teachers had difficulties in deciding if the utterances were pro-active or re-active.

I explained more carefully the ET table as well, and the observers used it and filled it out as the example shows (my translation):

Concept	Reference context	Signs/Symbols/Representation
Geometry	Area of the waste	2(5.25 x 5.25) + 2(15.10 x 5.25) = 213,675 cm2
Fraction	A quarter, a half	Calculator
Percent	The percentage of the waste	Calculator

Figure 35: A part of an IF table filled in by an observer

This made it possible to discuss mathematical expression and communication in plenary sessions, and also how the teachers expected or required that their pupils used their mathematical vocabulary. Furthermore, we had examples where we talked about what concept the reference contexts and the signs belonged to. In the plenary discussion, several expressed that it seemed to be useful for their practice to be aware of their own utterances, compared with how they wanted and expected their pupils' expressions to be. This discussion was held in connection with the lecture about the theoretical concepts.

During C-06 one observer (ob1), who used her own teacher focus is, observed solvers consisting of three women, called 'A', 'J' and 'M', who were all specialised in mathematics. One of the solvers (A) was a young teacher with only two years of teacher experience, while the other two (J and M) had more than twenty years of experiences as maths teachers. The observers' documentation of the interplay between them looked like this (my translation of her notes):

'A' just makes comments and quiet conclusions. 'J' asks investigating questions quietly, while 'M' is very dominating and takes the initiatives in a loud firm way, like: "It is the area we have to calculate" and when things come to a standstill, she takes initiative before it all stops. 'M' has the calculator, which gives her power because it is the only one in the group.

From another group, consisting of four people (D, E, C and S) 'ob1' wrote this documentation (my translation of her notes):

This group has one calculator to four people, it is a shame. E starts to outline the problem in the task, D works alone, and the other three work together. E uses D's paper to explain something to the other and all of a sudden everybody uses D's paper. In that way she is forced to take part in the work of the group. (…) Each person has a different role in the group: D writes every thing down, C takes notes, draws and makes the box in practice, E writes a little, S does not write at all. C and D argue about a result, C becomes upset, and after a while D admits that she was wrong.

Both of these observers looked mostly for how the cooperation among the teachers worked. In my opinion, they were very good at reading what went on in the groups related to the social norms. When they presented their observations in the plenary session, I asked questions about how it felt to be the teachers who solved the problem and were observed. This time we talked about what the observers saw, noticed and criticised. From the group with the three women, we discussed what happened when one from the group took the leadership and did not listen to the others. I know from the young teacher's headmaster, that this episode influenced her, but at the same time she expressed satisfaction with the course. From the other group we discussed how to involve everyone in the same communication, and the little trick of writing on another person's paper was highlighted. From this summary we discussed 'How to work together in a way that is beneficial for all?' The conclusion was that:

- Time was an important factor, as was
- knowledge about how to solve the task, and
- to know what kinds of problems the solvers had, and finally
- that it was sometimes OK to take breaks to work alone if necessary.

As a way to organise such group work, it was suggested, by a teachers and elaborated by the group, to start with a round of discussion, where everybody should explain how they would like to solve the task, and from that round make a plan for a solution. In that way this plenary discussion generated a plan for how to work with communication in a group that was supposed to solve mathematical problems. We debated whether this way of working called for a leader or a plan.

We discussed, furthermore, what could be done to change any habits the 'teachers' wanted to change. Several of the teachers expressed that they became aware of their own habits, like how they dominated others, or how they listened to others, and that they wanted to change some of their habits if possible. My answer was that I hoped it was a first step that they had been made aware of their habits; the next step was to find a way to change what had to be changed. This in-service course provided some ideas for how to do observations in other ways.

My presentation of some solutions of the task was done fast. When the 'teachers' saw the solution, they did not understand why they had such problems with solving it. In fact, not all of them had difficulties with the solutions this time. As for the openness of the task, they said it was this kind of openness which 'worked itself out of the clichés and legalised new kinds of interpretation'. One found that the task was not open at all, because the box should have a maximal volume.

In the lecture about the theoretical concepts, I repeated more or less the same lecture as from C-05. When I asked the 'teachers' how they imagined their pupils were able to conceptualise the mathematics and what kind of knowledge they needed to do so, they had several ideas. They realised that they had a kind of an epistemology in their own practice-theory – even though they did not know the word; but they were not able to express it clearly. It seemed that they had their own practice-theory as tacit knowledge. They expressed that they would use the theory as an analytical tool to observe and assess their pupils. The combination of the IF and ET concepts felt natural to the participants on C-06. The IF was closer to what they normally did, while ET was a reminder of what mathematics teaching is about: Learning mathematics.

In my log I wrote:

It was a long discussion, maybe a little too long for the teachers; they seemed tired at the end, as if they had had 'enough'. It is difficult to keep the steam up for so long. We had an interesting discussion, but their mathematical discourse was still difficult to discuss. When I showed the solutions, everybody finds the task easy, but when they work with it, they have difficulties. I need to find better ways to do that. The tables seemed to work better; at least now the teachers used them.

3.1.3 Discussion and conclusions of using IF and ET

In the official evaluation on C-06, the teachers used positive terms to express their experiences and benefit from the course. The following list comprises only few of the comments that the teachers made, but they were chosen to show the general trends (my translation):

- *Concrete procedures to plan teaching,*
- *Good open tasks for group work,*
- *New special knowledge input,*
- *Time to work with relevant issues,*
- *A clear connection between theory and practice the whole time,*
- *Good to have time to experience exchange between colleagues,*
- *Role-playing was interesting,*
- *I feel better equipped to assess pupils work*

In our last discussion on C-06, it was mentioned that quite a few were surprised to observe themselves and colleagues having the different kinds of difficulties they had. One of the 'teachers' on C-06 explained how she had expected another kind of course: She expected it to be boring and with much more explanation about what to do and how, but she realised that the course concerned her way of doing her teaching, and her ideas about how to develop this in a form that she could recognise.

Several of the teachers from C-04, C-05 and C-06 tried some of the ideas out in their own classes. Mette, who was observing the group (ET) on the C-05 course, described how the pupils now had a more focused discussion than was usually the case, because she used pupils as observers. She concluded that it was a positive experience for her, and that the strengthened focus on how to express mathematical concepts gave new inspiration and energy to the class. She did not provide more details from this work. The different testimonies told me that the in-service teaching inspired at least the ones who reported back to practice in news ways in their own classes. I did not observe them doing this, but I saw documentation in the form of pupils work and heard the teachers' testimonies. It is difficult to picture how the individual teacher organised her teaching, as I realised when I was observing teaching. But I had no reason for not believing their stories, even though I had only their words for it. I interpret them as their way of making sense and documentation of their teaching experiences. My documentation can be seen in the same way. I collected different data such as video- and audio-recording, log entries, solved tasks, materials produced during the course, pupils' work, evaluations etc. I have much more data than I can append here, which is why I make only short descriptions of some of the situations, and only when many similar examples have convinced me that I refer to a general trend.

The results from these workshops can be viewed from different perspectives. The observing teachers at the courses tried out a practice in a safe atmosphere, where they had the possibility to discuss what they saw with the people they observed. The observed teachers had the opportunity to look at themselves and discuss their strategies with others; maybe not all of them found it as 'safe' as the observers, but none of them spoke this out loud. The whole group of teachers was introduced to new educational methods, which they could use as analytical tools in their teaching. Parts of the IF and ET were transposed into practice during activities that the teachers carried out in the workshop. The focus for the workshop was communication and the goal was made apparent through the activities. The situation with the observers contained a feedback mechanism for reflection. It became clear to me that the teachers had difficulties with collaboration and also with the math task. The observers' comments enabled some of the solvers to face their own behaviour related to collaboration, and to discuss how group work and collaboration can be difficult and that it has to be taught. All of them realised that solving mathematics problems in a group is not easy. My intention was that the teachers should become aware, through their own experiences, of the difference between discussing a problem, that was not quite clear to them and when they had a clear understanding of the problem. They also learned how difficult it was to communicate when some of their partners just 'think out loud' instead of communicate. The observers expressed surprise that the communication that took place during the problem solving resembled the way their own pupils solved problems, and that collaboration was so difficult.

The conclusion for me as a teacher educator is that the IF and ET were eye openers and useable tools for observing interaction and development of how pupils use mathematical signs. The workshop stimulated the teachers' awareness of their look at development and how they demanded results from the pupils. To be realistic, it was only the beginning, because it was obvious to me that it was very difficult for them to use the tools: they only tried them once, and to observe in such a focused way was new to most of them. It is difficult to know for sure, but I realised that the guiding principles gave me a scaffolding and the 'teachers' a need and motivation to understand what kind of benefit they could gain from the theoretical concepts. In my interpretation, the combination of IF and ET accompanied each other very well; IF revealed how the communication took place and ET revealed the mathematical content. Some of the difficulties for the 'teachers' were that they looked for several perspectives at the same time, and even though they should look for mathematical utterances, they looked for social interactions as well. It is not clear at all how the teachers would use the tools back in their own classes. To reflect on one's own practice is both hard and difficult if it involves changing habits. In this situation, it was both fruitful and, for some, a defence. A workshop, on a short in-service course like this, can spark awareness in the teachers, but it cannot help us predict how their teaching will develop. In fact, their teaching is influenced by so many other things that it is difficult to say anything about causality. How to teach the mathematics teachers in a way that gives them the best competencies for their work in schools is not at all clear. As for the research, the practice is always more complex than the theoretical concepts. For me this meant that the guiding principles worked as a scaffold, but how to deal with the habits that were revealed was not clear to me as a teacher educator. The fact that the didactical contract was not apparent to the 'teachers' in the beginning was not reported as a problem, maybe because it was a part of the GP-teaching and therefore I was aware of the 'problem' from the beginning. It could be considered strange that the different 'teachers' still expressed their satisfaction with the course, when in fact they had more challenges than the in-service courses held before C-04, but they still had an enthusiastic teacher educator, which usually makes the atmosphere pleasant.

3.2 Virtual Monologue

I chose to use the Virtual Monologue (VM), introduced by Leron & Hazzan (1997), for the meta-didactical transposition in a workshop about communication on the in-service courses, because this article inspired me to reflect on whether such a VM took place at all and if it did what it was. These thoughts gave me the idea that the important issue was not whether VM was of this or that type, but rather how the 'teachers' created a VM if they did. The introduction I gave the 'teachers' on the first course C-04 was different from my lecturing about IF and ET: I chose to present parts of the article along with the activities. I give here a brief introduction to the research idea behind the VM and refer to this, when I describe how I used it on the courses.

VM is introduced as a reflection tool for researchers as a reproduction of the student's voice given as a monologue in the first person, in which one view of what might be going on in the student's mind becomes shareable.

> *We try to take the student's view by 'looking from within', by trying to create the student's mental state as best we can, and by trying to communicate our image of this mental state as faithfully as possible. (Leron & Hazzan (1997), p.269)*

VM is a tool, in which an experienced teacher or researcher uses the narrative mode to vividly convey his or her view of the student's mental processes (Ejersbo and Leron, 2005). Thus the VM is a tool that can help the reflective practitioner move from practice to theory through reflection:

> *…and find ways to evaluate and analyse mathematical tasks – that will help us reveal more of our student's reasoning powers rather than their weaknesses. (Leron & Hazzan (1997), p.267)*

The main idea in a VM is to try to understand the student's view of the world with the knowledge that it is of course not possible; trying, however, gives a kind of understanding. In most classrooms, we can find pupils that seem lost and confused when trying to make sense of the situation. These pupils react differently from pupils who easily solve math problems and their behaviour can be difficult to interpret correctly:

> *…but it may be approximately marked by contrasting it with affective and social perspectives. (Ibid, p.266)*

The traditional focus on mathematical understanding is a strong emphasis on cognitive aspects, with neglect of emotional aspects; the VM concept tries to challenge this focus.

The L&H article is based on examples, and in my workshops I used one of the examples with a task 'expectation' and interview with a pupil, Dina. I bring this task and interview here because I used it in this form (in Danish) on the course. The task and interview in the L&H article is from Sfard & Linchevsky, 1994, pp. 218-220 (henceforth abbreviated S&L). It involves the following task:

> The task (from S&L):
> Is it true that the following system of linear equations
> $$k - y = 2$$
> $$x + y = k$$
> has a solution for every value of k?

The expectations of the researcher are described as the following (from S&L):

In a problem like the present one, the objects that the students are supposed to consider are not just numbers – they are functions. To understand the question, one must realize that each of the equations, [...] represent a whole family of linear functions [...].

The interview with Dina (from S&L):

(Dina is a tenth-grade student, working on the above task; 'I' stands for the interviewer)

D: [reads the question silently] "... has a solution..."
I: What does it mean 'has a solution'?
D: That we can put a number instead of k and it will come out true.
I: When we say that the system has a solution for every value of k, what is the meaning of the word 'solution'? Is it a number or what?
D: Yes, it's a number.
I: One number?
D: Yes, it's the number that when you put instead of k, then the system is true.
[...]
I: This word 'solution' here, to what does it refer? Solution of what?
D: Of the equations, k - y = 2 and x + y = k.
I: What is a solution of these equations?
D: When we substitute numbers...
I: Instead of what?
D: ... instead of x, y, and k, and it comes out true.
I: So, once more, what are the solutions we are talking about in the question [points to the words 'has a solution']?
D: I think ... I think that I need three numbers: x, y, and k.

The interpretation of S&L was as follows:

Dina was helpless when faced with the problem. She asked the interviewer what she was supposed to do. The question was obviously not clear to her at all. After a minute or two of looking at the problem she said, "I am groping in the dark".

And, a bit later:

[She was] unable to interpret the question in a meaningful, consistent way. It left her confused and helpless. Thus, when asked what she was supposed to look for, she had no choice but to 'shoot at random' with pieces of standard statements which had worked in the past.

(For the complete discussion, including the VM analysis of the Dina interview, cf. L&H, Section 2.2.)

The next step in L&H's article is to create Dina's virtual monologue (here pp. 271-2; the italicized phrases are taken from the actual interview with Dina, as quoted above from S&L):

What do I have here? A system of equations... Oh, well, I know how to do that. You just have to solve it. It does look a bit different, but I can just do the usual solution. [reads the question silently] "... has a solution... for every value of k..." I don't understand this phrase. Why don't they just say 'solve' as they always do? I don't think we had this question before. So how can I solve it? What am I going to do? I really feel I am groping in the dark here. What does it mean 'has a solution'? I am not sure, but usually solution means that we can put a number instead of k and it will come out true.
I: When we say that the system has a solution for every value of k, what is the meaning of the word 'solution'? Is it a number or what?
I really don't know. I don't even understand the question. What was the question? "Is it a number?" well, what else could it be? I don't know. Oh, well... [performing a leap of faith] Yes, it's a number.
I: One number?
Of course, what else? I wish I knew where these questions are leading, I am getting more and more confused. But at least it seems from the question that I was right – it is a number. Yes, it's the number that when you put instead of k, then the system is true.

[...]
I: This word 'solution' here, to what does it refer? Solution of what?
What do you mean 'solution of what'? When we do equations in class we never have such questions. We just need to know how to solve them. What was the question? Solution of what? Of the equations, $k - y = 2$ and $x + y = k$, what else could it be?
I: What is a solution of these equations?
When we substitute numbers...
I: Instead of what?
What are the letters here? ... instead of x, y, and k, and it comes out true.
I: So, once more, what are the solutions we are talking about in the question [points to the words 'has a solution']?
I think ... I think that I need three numbers: x, y, and k.

I used this part of the VM-article for preparing and carrying out the redesigned in-service courses C-04, C-05 and C-06.

3.2.1 Preparing

For C-04, I created a meta-didactical transposition for the VM. I transposed the original idea, which was written as a research article to other researchers, into a teachable workshop for teachers. My plan was that the teachers should experience an emotional event that would then be used for analysing and reflecting on the products from this work, which was one of my guiding principles. I wanted to draw the 'teachers'' attention to the possible interpretations, both from the pupil's and the teacher's (or the researcher's in L&H) perspective. I hoped to discuss that such a VM could be visualised in many ways and examine how the 'teachers' interpret *the need to make sense* (p. 274) and the *need to meet expectations* (p. 275) from the perspective of both the pupil and the teacher in the interview. With this tool, I hoped that it was possible to discuss communication in new ways, such as why the teachers answered the way they did and how they thought about the pupils' mathematical thinking.

I adopted an empathetic attitude. I assumed that the teachers' values would be apparent through the way they expressed their empathy with the pupil and the teacher, respectively. This would highlight individual differences in how they view the pupil's possible mental processes during the solution process. For the meta-didactical transposition I took a different direction than in the original article; we should try to imagine what kind of mental processes took place in the teacher's (or researcher's) head as well. It is easy to criticise the teacher while empathising with the pupils, but the challenge here was to examine both the pupil's and the teacher's inner voices. In the design of the actual teaching, I wanted to work with various kinds of reflections; just to select the article on VM satisfied the first principle in my guiding principles.

I used the example from S&L with the task and interview, combined with VM as constructed by L&H. I translated the following into Danish:

1. The task on linear equations with one parameter,
2. The researcher's expectations,
3. The interview with the student (Dina) and its original interpretation, and finally
4. The authors' interpretation, as seen through their virtual monologue.

3.2.2 Practice

The VM workshop was tried out on C-04, C-05 and C-06. They will be described separately:

C-04: During C-04, the VM workshop was held in the second part of the course, after all participants had practiced the course they had prepared in the first week on C-04, back in their own classes. The following is a description from the workshop.

The first step was to present the translated task and interview with Dina. I used an OHP and asked if one of the 'teachers' would help me read Dina's lines. I played the role of the interviewer, and after reading this aloud, we discussed how they experienced the communication. They were very sympathetic towards Dina, maybe because the task was difficult for them as well. Then I showed them the interpretation of this interview (S&L) and some of them reacted quickly by saying:

- Dina's answers seem relatively rational and the interviewer seemed to stress her in a way that made it difficult for her to think.
- The interviewer plays the usual teacher's game 'Guess what the teacher is thinking'.

I did not ask them for the results of the task in the first place, but I knew that most of the 'teachers' were not familiar with that kind of task. The 'teachers' formulated their interpretation of the communication between Dina and the interviewer as a result of the way the interviewer pressed Dina. They expressed agreement with L&H's interpretation that the data was only analysed from a cognitive point of view. The 'teachers' claimed, furthermore, that the interviewer did not understand Dina; neither did she attempt to. They clearly expressed understanding of and sympathy for Dina.

After a while I asked them:

- Could we guess what was going on in Dina's head during the interview?

One of the teachers reacted immediately as follows:

- It irritates me that you ask that question 'what goes on in her head'. I have no way to know what goes on in the head of my 24 students.

I was a little surprised, but before I could answer her, one of the other teachers replied:

- Why does that irritate you? Don't we all guess when we communicate with the students? How do you listen to them?

After a little discussion about communication and listening, I ended with a quotation from Covey (1989): Try to understand before you want to be understood.

This utterance was meant to be relevant perspectives, both when the teachers communicated with the pupils and when they listened and engaged in this workshop.

The next step was a slide with the translation of Dina's VM. Again we used role-playing. I was the interviewer and two different teachers were Dina and her inner voice, respectively. After the reading, I asked:

- What are your comments? And why? What kind of feelings does this version generate?

The responses again came immediately:

- No, that is not what she thinks, she thinks…

Different suggestions now filled the room:

- I have three variables here, but only two equations, strange.
- What does the k do here?
- The k must be a letter like x and y – then I just have to find the value.

- Why does she ask that way? I am sure she wants me to say something special. What could it be?

It seemed that their own difficulties, which made them identify with Dina, stimulated their creativity. The plenary discussion revolved around what might have been going through Dina's head. Now it was time to find out how to solve the task for ourselves. We did that together and when everybody agreed on the solution, we continued.

I now turned back to the original interview and we read it again together, but this time I read the interviewer's questions with a voice which was empathetic and with understanding. Some of the 'teachers' started to laugh, maybe because they saw how their feelings influenced their interpretation. Now I asked them how the task could have been designed, why the interviewer asked the questions she did, and how they would have asked, if they had been the interviewer. It seemed that the 'teachers' faced other difficulties, because it was hard for them to find any answers. It was easy for them to identify with Dina, the pupil, but much harder to identify with the teacher (in this case the interviewer), even though it should have been natural for them to think like a teacher. It was apparently easier to criticise the teacher than to understand her. Maybe they felt hostile towards the interviewer because they had difficulties solving the problem themselves. Now it was time for work with a new VM.

Before the second session on C-04 the 'teachers' practiced a course in their own classes. I asked them to keep a log from these class courses and send me examples of 'difficult' communication from the maths classroom. I received the following communication from a teacher, who wrote that she found it difficult and asked what to do in such situation. I got her permission to use it for the VM work. From her log, I took this:

"The class has been working with the topic: "Mathematics in your everyday life". The pupils have to make their own questions/problems, which should be used as basis for their work. The pupils have difficulties making their own questions, and not all groups had their questions ready on the day we had agreed on. Most of the groups nevertheless worked satisfactorily with their questions. One group had major problems. They had difficulties talking about the problems and seemed to give up. They made some abstract questions about organising parties. They decided that one of them should try to look at the Internet for information they could use. In the mean time, I talked with the other pupils about how they could make their questions more specific, and more like questions which could be answered with the help of mathematics (T: Teacher, P: Pupil):
T: How will you work?
P: I don't know.
T: Look at the questions. Is there one or more things you want to specify?
P: I don't know.
She seemed recumbent and resigned. I suggested that we could try to 'compare beer and alcohol'.
T: What could you compare?
P: I don't know.
T: What about taking three different beers and compare them? They are easy to compare.
P: OK, if you like.
T: What kind of beer would you choose?
P: What do you mean?
T: Yes, for instance Carlsberg, Carl's special or a Christmas beer?
P: I only drink Tuborg.
T: OK, then we accept that, but what about the other kinds?
After a long time talking, we came up with three different beers
T: Now, how can you suggest comparing them?
P: The alcohol content.
T: Yes, and what more?
P: I don't know
I suggested she looked at the labels, the price, the packaging, etc. She wrote it all down, but didn't come up with any ideas herself. It seemed she was not interested or that she saw it as my task to ask the

questions. I suggested she should search the Internet and find what she was looking for, but it never happened." (my translation)

The whole group worked with this communication, using the VM. We read it all together, and I let the 'teachers' make quick comments. They said things like:

- *Why don't you tell her to go home?*
- *I know such pupils as well.*
- *How can we manage such kind of pupils?*

These spontaneous comments made it seem pointless in a first view to understand the pupil because it seems not to be a question about mathematics, but we used the VM anyway. We made smaller groups where some had the task to make the VM for the pupil and some for the teacher. When we met again, the 'teachers' had a different interpretation of what happened, and how they would respond to that pupil. The groups who worked with the VM from the pupil's perspective now found that the pupil was unable to see any meaning in what she was doing. Maybe the pupil was interested in working on the theme 'parties', but she did not know how, the 'teachers' said, and without this knowledge, mathematics was far away; when the teacher asked her, she froze. They said: the teacher had so (too?) many ideas that it was not necessary for the pupil to come up with any ideas herself.

The VM interpretation of the teacher provided new insights as well. The 'teachers' concluded that they normally wanted to be sure that their pupils all worked well, that they were active in one sense or another, that the teacher could leave an individual pupil alone and be sure that she worked. This was what they saw as the reason for the teacher to come up with so many ideas, instead of asking the pupil some questions she had to answer, the 'teachers' suggested. Several of the 'teachers' who interpreted this episode said that they recognised the first communication and it seldom gave any good results, but it came about automatically, because they felt stressed and had so many pupils to teach. We discussed how the teaching from the log could be conducted differently. First of all, we agreed that it was necessary to break the pupil's pattern, but if this pattern was a result of the teachers' habits, it might be hard to break. One conclusion from this discussion was that the pattern could be broken in several ways, but that it depended on careful attention by the teacher and an ability to ask more powerful questions. It became clear in the discussion that the VM changed the participants' interpretation of the pupil and of the communication, but also that the suggestions for more listening were given without anyone knowing how to practice them.

I made, therefore, a follow-up task right after the discussion. In groups of three, the teachers should interview each other about how to ask such questions of their own pupils. All three of them had to make turns as the interviewer, the interviewed and an observer, with only five minutes for each role. This little exercise resulted in outbursts as:

- *I thought I was a better listener, I don't have enough patience*
- *It is interesting to watch someone else*

After this little exercise I presented the VM as in the article, and we discussed the whole setting. The presentation was short, because we already had worked with some of the translated material taken from the article. The main idea in the article was to discuss influence from pupils' emotions, but in the workshop we also investigated how we interpreted the pupils' utterances and how the teachers' utterances could be interpreted differently. This workshop was about how mathematical communication could be understood; besides the content in the VM was an open task according to the process and the results of the VM.

C-05: I repeated the same workshop on C-05, but this time I put it in the first part of the course, before the 'teachers' practiced what they had learned in their own classes. To assist teachers to work on their own, we this time used a dialogue from a book (Undervisningsministeriet, 2001, p. 16); I can not tell for sure if it authentic, in the book it is called an example. The setting is a discussion between a teacher and two pupils at a Danish oral examination concerning percentages – an area the 'teachers' all felt safe about. Two pupils want to find out some results before a percentage was added to a number:

Teacher: Take care; it isn't as easy as you think.
Pupils: It is easy to find 25%. We only have to divide with 100 and multiply with 25.
Teacher: And what kind of results is that?
The pupils calculate and answer: 42.50
Teacher: Is this the results we want?
Pupils: We want?
Teacher: Yes. If you started with DKK 100 and then added 25 %?
Pupils: But we have DKK 170
Teacher: Yes, that is right, but if you add 25% to a number, and get the result 170, will the result if you subtract 25 % be the same?
Pupils: You have a point. Is it possible that you will come back later and now leave us for a minute?
Now the external examiner asks: Why can't we solve the problem together?
Pupils: OK, what would you suggest we do?
The teacher follows his previous idea and asks the pupils to add 25 % to a number they chose, and then subtract 25 % from the same number and see if they get the same number. The pupils select the actual number 170 and add 25 %.
Teacher: Now we have 212.50, and then try to calculate back to 170.
Pupils: How?
Teacher: Try to subtract 25% from 212.50.
Pupils: Wasn't it the 170…
And so the conversation continued. (my translation)

One could easily be condescending about the way the teacher asks questions, but working with a VM on this dialogue, a new dimension was discovered. The 'teachers' were split into four smaller groups: Two groups would create a VM for the pupils and the other two for the teacher. They were given 20 minutes to do this task. Then the 'teacher-groups' and the 'pupils-groups' presented their VM at the plenary meeting, which was followed by lively questions and discussion. Instead of only judging how the teacher asked and how he confused the pupils, the group that made the teacher VM tried to understand and identify with him. The questions they now asked were:

- *How was he caught in that trap?*
- *How could he come out of it without confusing the student?*
- *What kind of questions or comments could he make instead?*

Furthermore they started to reflect on their own way of asking questions, like

- *How do I ask questions myself and what kind of answers do I expect?*
- *Why are my questions the way they are?*

One of the 'pupils-groups' suggested that the students had a clever strategy for asking questions to the teacher, without answering anything themselves, a strategy they had not recognised before, but one they could now recognise in their own communication with pupils in retrospect. Working with VM gave the teachers time and opportunity to become aware of more details in mathematical communication. They were guided by their own emotional involvement and by the communication in the group. The discussion became different from what went on before: In my interpretation it was more balanced in the sense that the teachers tried to understand more than they made quick interpretations. The discussion contained more understanding and less criticism of the teacher, and this was different from their spontaneous comments; I felt it gave some new insight to all of us. As on C-04 I gave a final lecture about how VM was originally conceived. I did not get any transcribed communication back that we could work further with. From the logs the teachers kept between our meetings; therefore we did not work with any of their own communications.

C-06: For C-06, the workshop was planned to take place in the second part of the course; after the practice period. This time I wrote several letters to the 'teachers' in between the two parts of the course to encourage them to write down some of their own authentic dialogues and to send them to me. I got examples of communication from more than 50% of the 'teachers'. We used these examples in different exercises involving VM.

The workshop was conducted similarly to the workshops on C-04 and C-05, in the way I used the article. This time I had twelve dialogues from the classes. Not all of the dialogues were written out; some were rather a summary or a description of a dialogue. Often, the teacher remembered the overall outcome of the dialogue. I categorised the dialogues; it was difficult to be very categorical, but the headlines were 'quoted communication', 'referred communication' or just 'sentences' or 'answers', about mathematics or about other issues. I made every dialogue anonymous before we looked at it together and I did not show the 'teachers' my categorisation.

The following examples are from C-06:

1. Grade 9: "A task involved calculation of different areas on a football field and ends with the task: Calculate the dimensions of a field with an area of half a hectare. Most of the pupils read and understand this, and some did not. Most wrote 50x100m. One pupil wrote: 2x2500m. The area is half a hectare, but at a closer inspection we agreed that it would be a strange game of football on such a field."
2. Grade 8: "I never experienced that it was OK not to be smart at mathematics. Now, it became clear to me that mathematics is connected; I mean that the fraction is just a division and the opposite."
3. Grade 8: We worked with percentages; add and subtract them. My question was: A record shop sells CDs that cost DKK 139 each. You buy two CDs. The shop gives a 30 % discount on each CD. How much discount do you get on the two CDs? A pupil answered: "I get a 60% discount." Then I asked how much discount he would get if he bought 4 CDs. The pupil answered: "120%" I asked: "What does it mean to get 120% in discount?" The pupil: "hmm, oh I see then I will get money back if I buy the CDs, and I don't think I will get that." The pupil tried to understand that regardless how many CDs he would buy, the added discount would be 30%.
4. grade 8:the task concerns division with a fraction: 4:½:
T: You haven't written how you got the results?
P1: It isn't so difficult, it was just like this, and it is correct.
T: I can see that in the key as well and maybe you just took it from there?
P1: No, we haven't done that.
P2: No, we haven't
T: I want to hear your strategy for how you got the result
P2: Is it necessary that we write so much? Do we have to change it all?
T: Have you seen the rules for how to divide fractions?
P1: how is this done?
T: Look here, you divide a fraction in that you multiply with the reverse fraction.
It is shown

T: It is a great benefit if the tasks are complicated. Look for instance the next task: 4/5 : 2/5. Try to put them on a common fraction line and use the rule at the same time. Thus multiply with the reverse fraction.
P1: Oh, like that.
T: Write some comments in your formulae collection such as you are able to find it next time you need it.
It seemed as if they understood and could see the meaning.

I categorised the first two examples as sentences or answers to tasks, while the third and fourth examples was dialogues we could work with.

We discussed the difference between quoted dialogues and referred dialogues or sentences. We concluded that the quoted dialogues gave more information to work with than the referred dialogues. We agreed that it was necessary to look at the quoted dialogues to see what caused the outcome of a dialogue; where it succeeded or where it failed. The sentences had the same problem; we could not read from them what made the pupils say what they did. One of the 'teachers' said that it would be necessary to use a tape recorder, which I could only agree with, but at the same time we knew that it was not the way to work in everyday teaching – but maybe it could be used for certain special situations. Three of the twelve dialogues were used to make different VMs; one of them was example 3 from eighth grade about the CD's. In the presentation, all the groups at C-06 played roles, when they presented their interpretation of their VM for the others. Some of them made different versions of the VM for the same dialogue.

Example 4 concerned division with a fraction: '4 : ½', and the pupils had only delivered a result (T for teacher, P1 and P2 for pupils, VM1 and VM 2 for the different solutions):

T: You haven't written how you got the results?
P1: It isn't so difficult, it was just like this, and it is correct.
VM1: Why should it be a problem if we did it correctly?
VM2: I hope he will not find out that I just took it from the manual.
T: I can see that in the key as well and maybe you just took it from there?
P1: No, we haven't done that.
VM1: Why doesn't he believe us?
VM2: Oh, we need to take care, I wish he will believe that we didn't do it.
P2: No, we haven't
VM1: How irritating to be suspected to copy from the manual.
VM2: I hope it will succeed, I nearly believe now that we didn't copy it.
T: I want to hear your strategy for how you got the result
P2: Is it necessary that we write so much? Do we have to change it all?
VM1: How irritating if we have to make it all again, I don't have energy for that. We know how to do it, why shouldn't we then write it all down these tasks are easy.
VM2: Oh no, what now? I don't have the energy to write it all up, and I don't know if I am able to do it.

The VM goes on, but from this sample it is obvious how the group worked with two different versions, and we can not see from the teacher's questions which one is perceived to be more correct than the other. We had a brief discussion before the final lecture. I wondered why so few of the teachers used open tasks for the VM, but as they said, it was not significant for them that it should be an open task, because the way the teachers asked questions was more important than the 'openness'. The original article by L&H was sent to all the participants after the workshop.

3.2.3 Discussion and conclusion of using VM

The evaluation of these workshops could be viewed from different perspectives; from the teachers' point of view, from the teacher educator's point of view and from that of the researcher.

From the teachers' point of view, most of them expressed that they liked the game and that they had gained a new perspective on what it meant to understand a pupil. The goal with this VM workshop was to examine mathematical communication between teachers and pupil(s); in fact it was not to find out what the pupils thought, but to find out what the teachers thought the pupils thought. It was a discussion about how the teachers listened and how they understood what was going on. The first exercise from C-04 showed that the teachers gained new insights about the way they as teachers communicated to ensure that their pupils worked in an active way, not to primarily learn mathematics but to keep the activity going on. The way many teachers organise the work in the classroom means that the pupils need help at the same time and therefore the teachers feel stress and want to activate the pupils in a hurry. The 'teachers' on the C-04 workshop realised this inconvenient organisation and discussed how they could change it.

On C-05 none of the teachers sent me any dialogues that I could use for a VM. At this course, I presented the idea in the first part of the course. Some of the participating teachers gave a reason for not sending me a dialogue: One said that he felt it was too private, another that she forgot it; others sent me summaries from their logs, but without any dialogues. Others sent snippets of dialogues after the course, but it was too late to be of any use on the course. Maybe it was too difficult to repeat the workshop with their own classroom dialogues. Because of that experience I prepared the workshop on C-06 to be in the last part of the course. On C-06 it was different because so many sent me dialogues. The discussion on C-06 also concerned how to remember a dialogue with a pupil and for what reason. This discussion made the 'teachers' aware of different ways to present a dialogue.

A direct result of working with VM in this way was a change of teachers' way of listening to their own communication, and awareness about how they interpret pupils' utterances. The 'teachers' reported that they became more aware of how they asked questions and listened to the pupils after the VM workshop. At the same time, they also expressed more uncertainty. What they had been doing automatically before, now all of a sudden seemed questionable, and they had not yet developed on alternative behaviour.

The evaluation from the teacher educator's point of view was mostly satisfaction with the workshop because the 'teachers' had responded by making VMs, which gave rise to interesting discussions about mathematical communication in their classes. In that way the workshop seemed more or less as a success. The first introduction to VM as a role play worked in the same way at all three courses. On C-05, we could not use the teachers' own dialogues but it worked alright to use one from a book. However, I am sure it would be better to use some of teachers' own dialogues, but we did what was practicable. On C-06, the teachers learned how to memorise and reflect upon dialogues. I myself got new experiences about reflection, because the teachers in the first place had difficulties with choosing suitable dialogues and later on came up with so interesting and surprising virtual monologues. New methods were tried out for categorisation of communication in authentic situations.

The process was not entirely easy, and I often had to improvise. It was not possible to foresee what kind of discussion would emerge – a teacher never can. I had to be the reflective practitioner and reflect intensively in action. The workshop facilitator may feel a loss of control from having to deal with so many different 'teachers', and from being the one that had to decide what kind of feedback to give, what kind of summary to make, what the next step should be and what homework to assign. The energy comes from all the participants, but the facilitator has to provide the direction. In this way the situation itself contained a mechanism for reflection, one which I believe is an important first step in teaching and in learning how to reflect on communication in action.

To create a VM, or to try to express what a pupil or a teacher might be thinking and feeling, was an open problem that does not have one single solution, or even a best one. This brought up many feelings and ideas in the ‘teachers’, and gave them an opportunity to discuss what had come up. It was easy for them to express what they thought the pupil might have been thinking, easier than having to learn an abstract and detached theory. The VM started from their knowledge, from their understanding, from what they knew best and felt safe with. They could use experiences from their work life and acquired a tool for reflection in and on their practice. The emphasis was not on open mathematics tasks, but on mathematical communication and on why the discourse was what it was.

In summary: I found that the VM can be a powerful tool for reflection, but like all such tools it should be used with care and with an eye to its limitations and shortcomings. One obvious limitation is the subjective and ambiguous nature of any particular VM created in a specific situation by a specific person. A second and perhaps more serious weakness is the fact that I use a verbal medium to describe an essentially non-verbal phenomenon – the pupil’s mental state. For this purpose here, it was chosen as an analytical tool for the teachers to better understand emotions in communication and develop an analytical tool to reflect on their own communication.

The guiding principles were easy to follow in this case, where I made a meta-didactical transposition of a theoretical concept (VM) into an activity on the in-service courses. The transposition consisted of transforming a research idea into teaching activities for a group of teachers, and to decide what to use as materials, and how the teachers could practice on their own communication. The original concept was something to discuss among researchers; how evaluation of pupils’ utterances is often interpreted from a cognitive perspective instead of a more empathetic viewpoint. The main idea in my VM-workshops was embedded in the activities, in which the ‘teachers’ had to create a VM themselves. The activities highlighted emotions and held a feedback mechanism on reflection when communicating about mathematics. In this case, the teachers became aware of some of their tacit habits and routines. The goal was to make the teachers aware of their own mathematical communication and how it influenced the answers from the pupils. One of the teachers said that she was not patient enough to wait for an answer; she is not alone. In an English investigation of classroom assessment (Black and Wiliam, 1998, Black et al., 2002) it is demonstrated how often the teacher does not take the time to wait for an answer or to listen to unexpected answers; often the teacher answers his own question after no more than two or three seconds. In these VM workshops, the ‘teachers’ became aware of their own habits of asking (S1) and learned how powerful the influence of communication could be. The emotion stimulated their interest and they became emotionally involved and able to reflect in action. Measured by the positive reaction from the ‘teachers’ in all three courses, the guiding principles seemed to work as predicted. Minor changes were made for each new workshop, such as me being more precise about why we worked with this topic, and my own communication became clearer as well. The overall plan for the workshop was the same for all three courses, except the position of the workshop among the other course elements. This decision bore an influence on what kind of communication could be used in the workshop. If the VM-workshop was placed before the practice period, the teachers could use such reflection when they taught, but we could not work with the dialogues together; if, in contrast, it was placed after the practice, we could use a VM based on their own dialogues.

3.3 Socio-mathematical norms

At C-05, I decided to try out a new research concept in accordance with my guiding principles. The concept was 'Socio-mathematical norms' developed by Yackel & Cobb (1996; henceforth abbreviated Y&C). This concept also concerns mathematical communication. The reason for taking in a new theoretical concept was that I lacked a tool to make the 'teachers' aware of what they evaluated as 'best practice' in their pupils' work and why, which tied in with the epistemological triangle.

The following description of the meta-didactical transposition is brief, because I only want to show how the guiding principles enable us to take new concepts and transpose them into in-service education. First, I present a brief description of the concept, how I used it and how the 'teachers' reacted. The key issues are the process in which the activities revealed some of the teachers' tacit habits and their reactions to this experience.

Socio-mathematical norms are described as different from general classroom norms: as normative aspects of class discussions that are specific to students' mathematical activities; the latter is called socio-mathematical norms (SMN). The main idea with the SMN is to be aware of the mathematics communication when responding or discussing pupils' suggestions for solutions to mathematics problem-solving.

In their work with classroom observations, Y&C realised that what was valued, in relation to mathematical explanations differed markedly from one classroom to another and from teacher to teacher. The different mathematical norms were constituted by the teachers and the pupils in the particular classroom and in that way became part of the hidden curriculum, because it was not explicit norms, but rather an unconscious practice.

It is normal at all levels and in all topics to have common discussions that involve the entire class. The way this dialogue is realised is crucial for the teaching and learning. The SMN's in classroom discussions reveal how mathematical meanings are negotiated and 'taken to be shared' (Voigt, 1996) in the classroom discourse. Some results from studying the SMNs were:

> *… the process of negotiation of what counts as a sophisticated or an efficient solution was typically more subtle and less explicit than was the case for different solutions.(…) we note that the analysis of sociomathematical norms clarifies the process by which teachers foster the development of intellectual autonomy in their classroom. (Y&C, 1996)*

In terms of the didactical contract, I would assume that the teacher's norms are 'read' and followed by the pupils, and in that way the pupils know what is valued as 'best' answers without any special articulation by the teacher.

3.3.1 Preparation

The transposition of the theoretical framework 'sociomathematical norms' (SMNs) from the article 'Sociomathematical norms, argumentation, and autonomy in mathematics' (Yackel & Cobb, 1996) was prepared for and carried out for the first time on C-05 and again on C-06.

The three teachers that I observed in their classroom during the pre-study all had clear social norms for the pupils in their classrooms, such as raise a hand, speaking one at a time, behave properly and to do more or less what the teacher told them to. The pupils had learned to behave within social school norms. The didactical contract concerning their SMN was, however, not obvious; neither to me nor to the teachers; and as for the pupils, they mostly found their own ‘surviving strategies’ according to the specific teacher. The topic of this workshop was how the teacher could develop feedback to the pupils when their results were discussed.

The implicit assumptions, constituted by the way the teachers and the pupils act in the classroom, are very strong, not the least so because they are hidden. Making them apparent to the teacher through activities that causes reflection may give rise to anxiety, and therefore it has to be done with professional care. The purpose of transposing this theoretical concept was to enable the teachers to watch their own responses when discussing how they valued different mathematical expressions. The transposition consisted of activities that were initially only described as a research-based concept for use in a classroom. The aim was to set up discussions, which we could later discuss on a meta-level. In this workshop, my role was to be a mathematics teacher, and I deliberately played on the similarities between the two classrooms: the school and the in-service classroom.

The SMN-workshops were inspired by Clarke’s soap packing task (Clarke, 1996):

Soap-packing
You have a task that must be completed in two work weeks.
You will do the work at home. The job is to pack small bars of soap in boxes that will go to different hotels around the country. You will receive DKK 4 000 for the job, but you have to pay the transportation costs yourself.
You pick up the material on Monday, 4 July. First, you take the bus to the soap factory. Then you take a taxi home with the soap and the boxes. The bus trip costs DKK 20. And the taxi ride costs DKK 180.
After one week, you realise that you cannot complete the job by Friday, 15 July. You call Anna and ask her to help you. She can work Tuesday, Wednesday, and Thursday - about 30 hours all told.
When you begin working together, you discover that Anna work faster than you do. Anna packs an average of 150 soaps per hour and you pack 100 soaps per hour.
You finish late Thursday night. You and Anna take a taxi on Friday morning to drop off the soap boxes. The taxi ride costs DKK 200. You take the bus home and it costs DKK per person 20.
Two weeks later, you get paid DKK 4 000. How will you split the money between the two of you?
Make at least two different budgets that will show how the money can be split. Explain why you have split the money as you have.
Present the solutions on OHP transparency. The group has twenty minutes to solve the task.

The ‘soap packing task’ lacks certain sorts of information, which makes it a genuine open problem with several degrees of freedom. I expected that the participants’ solving strategies and how they performed would highlight norms that they were not previously aware of. I wanted to discuss the results and use this discussion as a reflection tool with the ‘teachers’. I wanted, furthermore, to discuss what they found distinguished the results from each other, and whether any of the results were more sophisticated or efficient than others.

3.3.2 Practice

The following examples are solutions from group works on C-05. The teachers had twenty minutes to solve the task and prepare their presentation. The following dialogue was taped during the solving process and shows how one of the groups discussed two different ways to argue for their solutions, paying by the hour or paying by production:

T1: One of the suggestions could be that we shared the DDK 560 and the other 15 500 in a 110:40 ratio.
T2: Yes.
T1: And the other could be that we divided the 35 into 140 hours and…
T3: Wouldn’t that be unfair to Anna?
T4: Paid by the hours or paid by production?
T1: Because she does the most work, you think?

T2. The other picked up the soap; the question is whether that counts as work (my translation)

This way of distinguishing between the solutions was used by the majority of the groups, but they referred to other solutions, and they discussed other related issues during the group work.

Assembled again in a plenary session, there was laughing from several groups, and I felt a pleasant atmosphere; I believe that the 'teachers' liked the task, which they also told me that they did. The task was to produce at least two solutions, and so with six groups, we had twelve solutions on six OHP sheets. Because the groups presented their two solutions on slides, it was easy to see the different solutions.

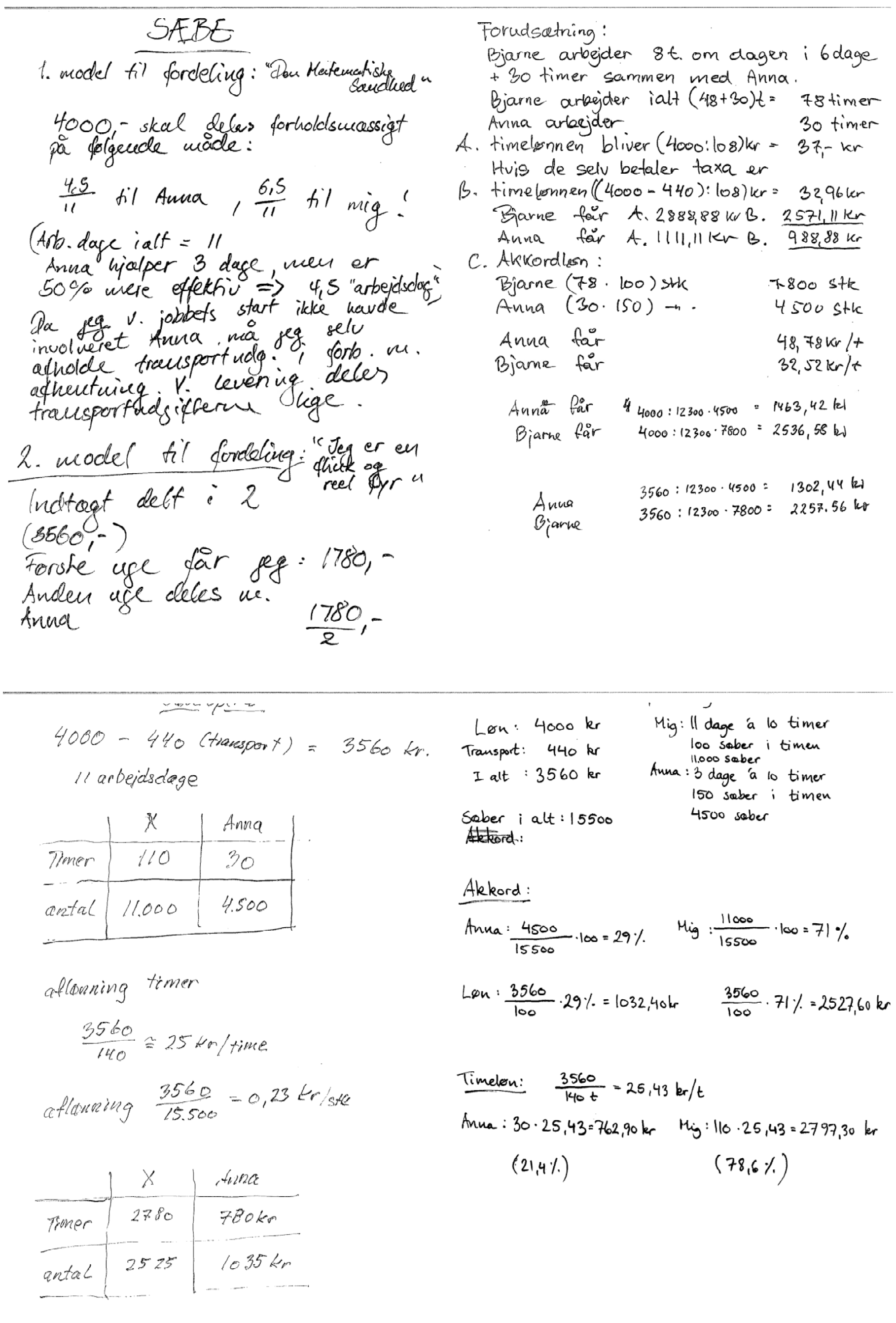


Figure 36: Examples of solutions that show different ways to calculate and present the work

The presentations were brief, but I noticed that several of the groups started out by saying that their solution was the same as the other groups, even though they had other results for instance because they had not calculated the payment for each of the workers in the same way, they had used different calculation methods, or even decided on different hourly wages. What they called similarities, I realised, was the principle between dividing the payment as payments by the hour or per production. As for the discussion, I copied all the solutions for each 'teacher'. In the following discussion, we examined differences and values.

It seemed that some of the teachers disliked this discussion:

- One said that he couldn't care less if only their pupils had an acceptable solution.
- Another said that the task was to give two solutions and not a presentation.

But others again were curious about what we could learn from this experiment; they drew attention to:

- How easy it was to read and interpret what the group meant, we could call it the 'communication value', or
- How sophisticated the calculations were.

Based on such expressions, we discussed what approaches were more sophisticated than others, how important it was to be unambiguous as a teacher, which I felt was directed at me. The aim with the discussion was to find out how conscious the 'teachers' were about their use of SMNs in a 'taken as shared' discussion.

When the 'teachers' solved the task, they understood it in one way, but it was a different song when I asked for an evaluation of the results. They differed in the way they interpreted how the results should be presented. The sheets were different as can be seen in figure 36: Some made only short notes and a more detailed oral explanation, while others did more writing.

The discussion about the 'best' presentation made a few of the 'teachers' defend themselves in an unfruitful way, while others expressed that this discussion enabled them to distinguish between different presentations. They agreed that it was new to them to discuss minute differences and similarities. At the same time it was a balance for me as the teacher educator to know how I should assess their calculations or let the discussion about the results goes on on equal terms. I had this problem because the solutions once more resembled pupils' solutions, and it could endanger the whole workshop to go into this discussion.

We finished the workshop by reflecting on all the processes; solution of the task, the presentations, and the discussion of the solutions. Finally I gave a little lecture about the article and the socio-mathematical norms as Y&C defined them, and I pointed out the importance of taking responsibility as 'a representative of the mathematical community' as Y&C call it. The teachers expressed that they took the SMNs very seriously, and that it was difficult to distinguish between qualities and to give the 'right' response on the fly in the classroom; on the other hand they did not show a big interest in the article; I left it on a table and asked the 'teachers' to take it if they wanted to read it, but only a few of them did that.

On C-06 I repeated the workshop with minor changes, such as being more precise about how to give the presentation. The discussion we had on C-06 went livelier than the one on C-05, which could have several explanations. On C-06 several of the teachers said that they did not normally pay attention to how they discussed results with their classes, but this new method was worth trying, because, as they said: it seemed important and they were not very good at it. They also said that they lacked tools to evaluate the results that came from their pupils in other ways than deciding whether they were right or wrong. As for the article, some of them looked at it, but concluded that they would not read it, and they left it.

3.3.3 Discussion and conclusion concerning the use of SMN

This evaluation involves several points of view: The teachers', the teacher educator's and the researcher's. The teachers were predominantly positive; even though this workshop started with a task they liked and ended up showing some of their shortcomings. Some of the 'teachers' defended themselves by saying that it was not clear that I would use the presentation the way we did with SMN. The discussion made different points of view apparent. More than half the participants at C-06 mentioned this particular workshop as interesting in their final written evaluation, because, as they said, it connected theory and practice.

My role was double here, because I could try to do an exemplary teaching that was practicable, or I could keep the discussion on a meta-level. In the first way, then, I would have to demonstrate my values and be a good representation of the mathematics community, where I evaluated the 'teachers' solutions. Or I could move on to a meta-level way and arrange a discussion between more or less equal partners about how to discuss complex results or communication values. I mixed the two roles, but I never discussed this with the teachers. In the beginning I asked them, as a teacher would, whether any of the results seemed better or more effective than others. And after a brief discussion, where I felt that some of the 'teachers' were very defensive, we returned to a meta-level discussion. My questions were whether they saw any differences or similarities in different views, and how it was possible to evaluate this and how they would do it in their own classes. The danger with this form was that I noticed that some of the teachers felt awkward at being treated like pupils. I understand and find this danger a challenge for the teacher facilitator. Even though the situations in many ways are similar, it is unpleasant to be the cause of such an emotional reaction. If a pupil in school reacts that way, it is easier to find out what the cause is, because in a school class the teacher and the pupils normally know each other very well.

As for the researcher using the guiding principles in this case, I wanted to determine whether it was possible to work with a new theoretical concept transposed into practice. The emphasis was how tacit habits or knowledge became apparent and how this provoked the teachers. It was easy to solve the 'soap packing task' and it was a lot of fun, judging from the laughing and talking in the presentation, but when we tried to use the results for another purpose, the teachers no longer found it fun. Some of them found it necessary to defend their solutions, and would not accept any critical comments about them. The solutions became important to the ones who had made them. This came as a surprise to me; I had thought it would be easy to look at the solutions as different examples for the discussion. But on closer inspection I see that as long as I play the role as a teacher, the 'teachers' 'play' or are pushed into the role of pupils, and they behave accordingly. To work with tacit knowledge and reflect upon it comes at a risk, but at the same time our existing knowledge influences how new knowledge gets embedded. I see this part of the activities as a difficult but very important part of the guiding principles.

The conclusion from my interpretation is that the guiding principles showed both strengths and difficulties when they were used on a new research concept.

4. Summary of the main study

The research question concerned to what extent a meta-didactical transposition for mathematics educational research concepts can be incorporated into successive stages of redesigned courses, and how effective these redesigns might be, measured by the participating teachers' reactions during the course. The methods used for the transposition were my guiding principles:

1.
Each teaching sequence on the in-service course is based on a particular theoretical concept, framework or set of results found in research literature.
2.
The theoretical concept is transformed into practical activities for participating teachers, and the activities should accord with the following principles:
a. teaching objectives make the theoretical idea apparent,
b. teaching methods are practicable in the teachers' own practice,
c. activities make the teachers' tacit knowledge apparent, and
d. activities provide feedback mechanisms for reflection.
3.
In the preparation phase, theory precedes the designed activities. In practice, activities are presented before the theory.

With these principles, I held three cycles of the course, as presented in this dissertation. The descriptions, along with examples from these successive courses concerning the teachers' reactions, are presented in each of the presentations of the different theoretical concepts transposed into teaching.

The four theoretical concepts; Interactive Flowchart, the Epistemological Triangle, the Virtual Monologue and the Socio-Mathematical Norms, all concern communication and reflection in mathematics teaching, and were selected to assist in the design a meta-didactical transposition.

C-04 was the first course in the series that was redesigned according to the guiding principles, while the next courses, C-05 and C-06, were mostly just refinements; the socio-mathematical norms were a late addition. We can say that these guiding principles belong to the 'macro-level' for teaching, while the corrections made by the teacher educator (and observed by the researcher) can be seen as taking place on a 'micro-level'. In my adjustment of the courses, several observations were used for guidance, observations concerning the teachers' reactions to the activities, the discussion and how they responded when some of their own habits became apparent. If the teachers felt secure enough to discuss their habits, I considered myself to have 'worked better'; not in the sense that it caused less provocation or disturbance, but because of the importance of building up a 'safe' environment in which failure is acceptable and teachers feel comfortable making 'half-baked' comments. For the same reason it was important that the instructions given by the teacher educator were clear so they did not cause any uncertainty, which could be a source of irritation in the teacher group. Furthermore, it was important that the teacher educator was aware of her own part as a kind of role model in the discussions.

One crucial point, from the researcher's point of view was to investigate what caused the reorganisation of the redesign on a macro-level, and how the teachers reacted to the activities and the theoretical concepts, and how these reactions could be identified, interpreted and combined with the relevant criteria in order to make the course 'better' and more effective. It could be argued that the teacher educator should evaluate and decide whether anything should be changed, while the researcher observed what produced the reactions and the changes.

The activities contained challenges, which had in common that the teachers needed to use existing knowledge and would at the same time be made aware of their own habits and automated behaviour or beliefs through different reflection methods. The activities all included a kind of role-playing, where the participants had different positions, as observers or task-solvers, or they were all 'pupils' or were to play a person's inner voice. The benefit of this role-playing was that the teachers could be themselves, but pretend to play a role, which would perhaps be less 'dangerous'. The teachers reacted differently during the reflection when they became aware of their own behaviour or habits. I distinguish between two different reaction patterns: defensive versus exploratory. Some teachers became very insecure when we discussed them or their behaviour. The process elements described here could maybe be used for other topics than mathematics, but the content about mathematical communication, and the discourse we use to teach with open problems in mathematics, has influenced every conclusion made in the study; particularly the conclusion that the teacher's mathematical skills play a crucial role.

One example of an insecure teacher's reaction is from the first presentation of workshops, where one of the solvers was told that he was not listening to his partners, and he explained that he played along and acted impolitely, but that he knew it and was aware of it. He formulated this defence in such a way that it was difficult to discuss it any further; in a way he disarmed the observers by going in to their criticism. In the same case one of the observers reacted the opposite way: she had some difficulties observing what mathematics was used and admitted that it was a new and difficult task to observe others solving a mathematics task. Back in her own class, she tried out the same activity by letting her pupils do the same observation. She changed her teaching behaviour, because she found flaws in her previous teaching behaviour. In the same case we saw that the observer without a certain focus tended to look for the social behaviour in the group work more than for the mathematical expressions; I noticed this behaviour in all groups on all courses. The explanation given was that it was too difficult to observe both topics at the same time. This case showed us both defensive attitudes and exploratory teachers, who reacted to their own inappropriate habits and wanted to change it. It demonstrated, furthermore, skills and strategies in use for solving such a task. The teachers used the same strategies as pupils in their classes would have done.

In the case with the virtual monologue, one of the 'teachers' expressed irritation of the fact that I would even ask such a question so as 'to guess what went on as an inner monologue', while the others were curious and went along with the game and reflected upon their own way of listening and responding to the pupils in a mathematical context. In the case with the socio-mathematical norms, the discussion turned to how the task was understood, and that I broke the didactical contract by asking new questions.

My conclusion from the 'teachers'' reaction is that the activities made some tacit habits apparent, and the common reaction to this was either to defend their position or to accept that their habits could be changed. I believe the defensive reaction to be automatic and as the opposite of reflective. To be open to new skills, you must be able to reflect upon old knowledge or existing habits, because these habits or beliefs can influence the new skills. One problem is that we think and react automatically (S1-reactions) when we ought to use more energy to reflect upon ourselves (S2-reactions). My conclusion concerning the 'teacher's' mathematical skills is that their mathematical skills and strategies for solving the tasks correspond to those of the classes they teach. I believe that they are good at the mathematics they learn from the textbooks they use, and because they use textbooks so much, they forget other mathematical concepts. Working with open tasks in mathematics is a way to work with mathematical ideas, but this did not interest the ‘teachers’ when they observed or solved the task. When the observers presented their observations, their discourse about the open mathematics tasks did not involve much mathematics; the social processes seemed to hijack their attention.

As for the theoretical presentations and the articles, which I gave to participants in continuation of the lectures, they expressed satisfaction with the presentations, but did not show much interest in any of the articles. I translated pieces of the article about the virtual monologue, and this article was showed the most interest, but the teachers did not read any of the articles. From this I conclude that the redesign was a pathway for the teachers to encounter theoretical concepts.

The core of each theoretical concept was communication about mathematical problems. The final conclusion about the guiding principles is that they worked in the teaching for the activities, maybe not exactly as I had planned or conjectured, but in such a way that the teachers worked with topics that were difficult to them, and we also brought to light some of their tacit habits. To design and carry out the activities was sometimes difficult, but it was very engaging when it succeeded. In that way there were high spirits on the in-service courses most of the time, and the ‘teachers’ said that they found it interesting to look at their teaching from new perspectives. Based on this, I would conclude that this kind of meta-didactical transposition did give the teachers an opportunity to meet theoretical concepts – even though the interpretation of the ideas was my own - that could be used in their own practice, and the guiding principles could be used by others, both teacher educators and teachers. Meta-didactical transposition can be run in many ways, my guiding principles was only one such way. It requires further research to determine whether and how the teachers used these skills in their own teaching after the course; in the main study, I only followed them during the course.

5. Final conclusions of the dissertation

This study began as an investigation of an in-service course concerning teaching mathematics with open problems (Part 'Pre-study'), but eventually turned into iterative design research (Part 'Main study'). In the process I realised that the teaching on the in-service course did correlate with the expected outcome. I found it particularly necessary to teach how to develop communicative and reflective skills; for that purpose I used some research ideas from the mathematics education research literature. The ideas required a kind of transfer to make them teachable on the in-service courses, a process which I have called meta-didactical transposition.

In the course of this study, I ran into two kinds of dilemmas, which tended to create frustration both for me and for the participating teachers.

1. The teachers' goals did not match my goals: Often I realised a discrepancy between the teachers' expectations and my goals for the course; the dilemma was what expectations to satisfy and how and why.
2. The teachers' good response and positive feedback to the course did not match my observations in their classrooms; what they expressed about what they thought they learned from the course differed from what I observed when they taught in their own classrooms.

These conflicts had a major impact on how I planned and carried out the redesigned courses.

I tried to deal with these conflicts through the teachers' reflection-on-action, reflection-in-action and the meta-didactical transposition mentioned above.

The rest of the conclusion section consists of two subsections. Subsection 5.1 contains a concise summary of the research results from the pre- and main-study, and subsection 5.2 contains some tentative thoughts about practical applications of the research.

5.1 Results of the research

The pre-study: The pre-study concerned the in-service course about open practical problems in mathematics that took place in 2002 (C-02). The investigations focused on the extent to which and in what ways some of the participating teachers adopted teaching with open practical problems in their own classes. I looked for any problems the teachers might have had, and compared then with how those problems were addressed during C-02. The goal with this investigation was to collect data from which an evaluation and a redesign of the course could be made.

The pre-study revealed that, even though the teachers reported satisfaction with the course I saw that they used open problems in a problematic and limited way. The use did not meet the standards I thought the course would teach them. Analysis of the data has revealed explanations for this discrepancy.

One reason might be that the teachers' mathematical knowledge was not sufficiently developed. This explanation refers not only to their knowledge of mathematical content but also to how to transform it into teaching. As Ma (Ma, 1999) explains in her book, it is not only a matter of how many years of mathematics education the teachers get, but rather how well they understand the mathematics they teach and the way fundamental mathematics can be presented to pupils. In this case the use of open problem-solving for practical issues was not only meant as a new activity, but rather as a way to explore mathematical ideas, solving processes and multiple results in more details.

Another reason for the conflict was that the teachers' communication skills, both concerning mathematical communication and other communication, were not developed in a focused way. It is one thing to see and know the difference between mathematical communication and other kinds of communication, another to develop sufficient awareness and knowledge to ask powerful questions and especially to understand how different questions produce not only different answers, but also influence the pupils' understanding of the meaning in their work.

Finally, the in-service course did not prepare the teachers sufficiently to improve and recognise the competencies they needed to learn to teach mathematics with open problems. Teaching mathematics with open problems turned out to be harder than I had expected. Before the pre-study, I was not aware of the kind of difficulties the teachers had after they were introduced to the concept of practical problems with an open approach. The teachers came to the course to learn how to produce tasks for the final oral exam in mathematics and how to run this exam. The official requirements include a recommendation to formulate the tasks as 'open practical problems' and it is mentioned that the communication during the exam is important. Yet, I was not aware how important it was to design teaching activities that taught communication and how to reflect on communication as specific topics. I took for granted that the teachers mastered these competencies, because they mostly expressed enthusiasm about the course and what they learned from it. They were not aware that something was missing. I realised through my observations that open problems were often too open, and had no clear structure for what to solve and how. It was not clear what mathematics could be learned from these open problems. This openness and the missing structure meant that the teacher had to accept anything, because it was not possible to correct it or develop it in a mathematical perspective. It is not my intention to blame the teachers for lacking these skills; they never learned because they were never taught. Now they enrolled in a course and again they were not taught about what was so crucial for working with open problems: how to communicate and guide the work into an understanding of how mathematics could be used as a qualifying factor to reach useful results.

On C-02, I was not as aware about the discrepancy of the expectations as I am now. I was guided by my intuition about the activities and the teachers' expressed satisfaction; naively I expected that they more or less learned what I expected them to learn. From their responses, I interpreted that they did. In the observations, I realised that I had been wrong, and when I reflected upon C-02, I realised that I met the participants' expectations more than the goals for the course. I built up an environment filled with activities, but without awareness about how they adopted these activities. The most difficult I left for the teachers and expected that they somehow were equipped to handle the difficulties through reflection and communication. In the first place I met their expectations, and they had fun and positive experiences from the course, but their experiences did not match my ambitions for the course. Being aware of that, I planned the next courses to be more in agreement with the goals I expected, formulated in competence terms.

Conclusions from the pre-study formed the basis for the redesign of the in-service course. The important question that arises from the analysed data from the pre-study was whether the redesign could actually make any positive difference for the teachers' educational practice with open questions. The questions can be split up into:

- Which of the findings could actually be attributed to C-02?
- How could different teaching address the real problems?
- What are the alternatives?

The methodological tools used for the findings and the analysis of the data were qualitative in form of interviews with teachers, video recording of classroom teaching, logs from teachers and myself. I used ethnographical methods for the analysis and interpretation of these data, because the data held a lot of complex information. There were both weaknesses and strengths in these methods. The advantage was that I was able to work with 'unstructured' data, which I had to code in new categories. This work produced a new insight from an area that I was already very familiar with, and this gave evidence for my conclusion. The familiarity can be considered a weakness as well, because when I was both a teacher and a researcher in the same course, I had a tendency to notice and pay attention to how I myself understood teaching and learning, even though I wanted to make sense of the ideas and actions behind the teachers' understanding. It was a long and hard process, and it became a part of my own learning process. Two other researchers who have studied their own practice are Ball (Ball and Bass, 2000), (Ball and Bass, 2003) and Lampert (Lampert, 2001), who inspired me to believe in the project; both of their studies are about teaching mathematics in elementary schools.

The dilemmas, I realised in the pre-study, were the main reason for how I created the redesign and systematically changed it through several cycles in the main study.

The main study: The research concerned the redesigns of the successive courses and the teachers' reactions during the courses. The research findings revolve around the effectiveness of the meta-didactical transpositions realised through the GP-teaching and investigated relating to the teachers' response to the GP-activities and the use of various educational research concepts.

The research question was:

To what extent and in what way can a meta-didactical transposition be incorporated into the successive stages of a redesign of the in-service course, and how effective is this redesign, measured by the participating teachers' reaction on the courses?

The keywords in the research question are the 'meta-didactical transposition', 'effective' and 'the teacher's reaction'. I take these terms one at a time and summarise an answer to each of them.

As for the meta-didactical transposition, I designed the guiding principles, which are:

1.
Each teaching sequence on the in-service course is based on a particular theoretical concept, framework or set of results found in research literature.
2.
The theoretical concept is transformed into practical activities for participating teachers, and the activities should accord with the following principles:
- *a. teaching objectives make the theoretical idea apparent,*
- *b. teaching methods are practicable in the teachers' own practice,*
- *c. activities make the teachers' tacit knowledge apparent, and*
- *d. activities provide feedback mechanisms for reflection.*

3.
In the preparation phase, theory precedes the designed activities. In practice, activities are presented before the theory.

My answers to the research questions concern to what extent I could use these principles in the preparation and in the teaching practice, and how effective they were in relation to the aims of the teaching.

As for the effectiveness, it was only measured through the teachers' reactions on the course; through their comments and behaviour. This means that most of the interpretation of their reactions passes through my 'filter'. I document as much as possible with video and audio recording, logs and other materials, but also I describe situations that I do not document in detail because I have so many hours of empirical data. I attempt to describe the atmosphere, which I cannot document either, but I hope that the reader will understand what happened on the courses. As for the lasting importance, it is my hope that the transposition of research concepts should not only work on the courses. I hope the chosen concepts will work as a catalyst for the teachers' own practice after the courses, and that the GP could be generalised to work for other in-service courses as well. I leave to other research projects to investigate if and how the GP-teaching on the course influenced the teachers' teaching back in their own classrooms.

The research concepts were not developed specifically for the teachers, but rather as research directed to other researchers. Yet, in the transposition of the research ideas/concepts into teaching activities on in-service courses, they became practicable for the teachers, which made the concepts available first in the in-service classroom and eventually also in the school classrooms. This aim with the transposition was the same for all the research concepts. The transformation gave the concepts a new perspective, but changed it at the same time in a way, which was not mentioned or included in the original research concept. The research was now transformed into a tool for teaching, where a real challenge was to keep the main idea of the concept intact. It is an open question whether we can still call it the same idea. The research introduced here, all concerned tools for communication and reflection, which were the topics on the in-service course. It could be seen as a criticism that the concepts, which were meant for other researchers, were now transposed into teaching (maybe beyond recognition?), but it could also be seen as a strength that the transposition brought the research from theory into practice and in that way obtained another value. It depends on how much the original idea was changed in the transposition. A transposition will always change a concept, a question is how much and for what purpose.

One of the dilemmas is that research reported in mathematics education journals often has little relevance for the teachers' practice (Lester, 1998), because teachers and researchers have different discourses about what they know and need to know about mathematics teaching and learning; teachers use their developed intuition to describe teaching (moreover much of this knowledge is tacit), while researchers use an academic discourse to describe methods and validity, a discourse which is explicit. My intention was that the teaching on the courses should help the math teachers understand concepts from mathematics education journals/articles. For that purpose I needed the concepts to be more 'user-friendly'. The question is whether it is possible to change a concept to make it usable and still call it the same concept. In each final lecture, I used the original journal/article, and gave a copy to the teachers. In these lectures, I made the connection to the activities they had just gone through, and made it clear how I had prepared the activities based on the research. The guiding principles were meant to address the teachers' problems, to help them understand what kinds of skills they needed and to facilitate their access to theoretical concepts that contains some answers and analytical tools for these issues. My hope was that the teachers afterwards would read the articles and be able to use the source directly. This failed totally; none of the teachers read any of the articles. On closer inspection I admit that it was a naïve hope. When I ask my research colleagues, even they admit how difficult it is to find time to read articles after a conference, so how could I expect teachers to overcome that practice?

As for the third guiding principle, it has two implications: first, that the theoretical concepts should inspire the activities on the in-service course and be presented to the teachers through activities in such a way that the teachers realised the connection between theory and practice; second, the original source of the concept was lectured to the teachers in a way that the combination of first activity and then theory should help the teachers to adopt the theoretical idea to be used in their own practice as an analytical tool.

The teachers' reaction to the redesigned courses can be divided into two groups: defensive and exploratory. It is reasonable to assume that the defensive reactions occurred when the teachers felt threatened in their habitual routines, where they normally felt successful. It seems quite natural to defend habits that normally worked. Yet, the learning process was made harder with this behaviour. I see the teachers' defence mechanism as an attempt to avoid being confronted with their insufficient of mathematical skills and didactical competencies. When this insufficiency became apparent, the teachers reacted with replacement activities, as when the observer continued to praise the calculation made in the group work. There was clear evidence of this in the video from C-04. I will point out once again that this is not to blame the teachers; it is hard to avoid falling in the trap just to master the pupils' curriculum. If pre-service teaching does not give help to connect mathematics with didactics, pedagogy or education, and teaches how to work with mathematical concepts, then the teachers do have little to lean on. This means that the teachers, when they start their practice after pre-service education, mostly rely on the tasks from the text-books and experiences from their own past. In the school system, the main aim is to teach the pupils to solve tasks such as the final exam requires. The criteria for success are that the pupils get high marks, which seems to be the same as to master the tasks, in which the mathematical concepts are hidden. Making these mathematical ideas apparent depends on the teacher's skills and competencies. The teacher should be aware of mathematical concepts and possess the skills to ask questions that demonstrate the concept so that the pupils become motivated to work with it. These competencies are not what I see the most, when I observe teachers working.

The teachers who reacted with exploratory actions had the courage to examine at their own behaviour and to reflect upon patterns in it, which gave rise to fruitful discussions. On the courses, there were many of these exploratory behaviours, but the defensive behaviours have a tendency to dominate the process.

The validity and interpretation of my data can of course be discussed. The investigations I made to answer my research question gave me new insight into an area, which I am very familiar with, but never considered before in terms of how the teachers managed to teach mathematics with open problems or how they used mathematics themselves. As mentioned earlier, I used both my intuition and academic methods in my study. It is difficult to document intuition in detail, but I hope my work will convince the readers. What I have shown in this dissertation is for me valid documentation for some of the problems the math teachers meet in their teaching. My overall focus has been on communication and reflection when we teach open practical problems in mathematics, but there have been situations during the in-service courses where we worked with other topics that might have crucial influence on the teachers' practice: we worked with mathematical modelling and application, assessment of a mathematics oral performance, pupils' mistakes, intuitive representations and textbooks. Each of these areas could be objects for further research as well and have been, internationally.

5.2 What I have learned: Final speculations

Teaching mathematics with open problems is both vital and difficult. It is vital because we know from research that teaching with open problems gives pupils opportunities to work with the same problem on different levels and in that way makes learning/teaching differentiation possible, and it provides an opportunity to work with mathematical ideas in different investigatory ways. Before the study, I thought that the predominant difficulties were the weak definition of what an open problem is and a resulting uncertainty about how an open problem should be formulated. My understanding at that time was that if only the teachers had a clearer understanding of different kinds of open problems, and more experience with different ways of working with such problems, they would be able to use open problems in their math teaching in a productive way. The teachers seemed to think the same, because most of them expressed enthusiasm and desire to work with open problem-solving in their classes. Yet to my surprise, when I observed their teaching in my studies, I became aware of other difficulties the teachers had, when they worked with open problem-solving in their classes. And these other difficulties are the topics of the following pages.

Some of the other problems were that the teachers had a tendency to formulate the problems so openly that ‘anything goes’. This openness meant that the teacher did not know how to direct the pupils’ solutions in a mathematically developing way. One reason for this way of formulating open tasks was of course that they knew too little about what open problems could be and about the potential in using them. Yet, another reason was that to work with open problems in mathematics requires knowledge about mathematical concepts and about how to use them in the teaching; it was competencies the teachers did not have. The teachers had to be made aware of their shortcomings if they wanted to use open problems in mathematics, and not leave to the pupils what they could not manage themselves. The most significant difficulty I noticed when I tried to teach the teachers to use open problems in their classes was that the teachers were not aware how much their tacit beliefs or habits (routines) influenced their ‘new’ teaching. When I observed the classes, I realised that to use open problems was more complex than just to formulate a few tasks. It turned out that the success of open problem-solving hinges on how the teachers communicated with the pupils, and here I saw the real difficulties: How to ask questions and listen for answers combined with an awareness of how the questions influenced the pupils’ work. During the observation and interpretation of the data, it became clear to me that the teachers did not have valid strategies for how to achieve what they expected from their teaching - just as I did not either in my first courses. The teachers did not know how to let their aims or plans for the teaching come about in practice. They had ideas that clashed with beliefs or habits, which they did not notice or were not able to externalise. They expressed more satisfaction with their teaching than I felt was justified. I noticed that it was still a problem to ask open questions if

- the belief was and continued to be that an open problem involved having an opinion about an authentic problem instead of a question for a systematic mathematical investigation;
- the teacher was afraid of the syllabus and it therefore influenced the way he listened and required restricted calculations;
- the teacher was not certain about what kind of answers she wanted back and therefore she had to accept anything;
- the teacher did not possess sufficient mathematical and didactical competencies.

All this behaviour stemmed from the teachers’ habits or competencies more than from their aims with open problems. My conclusion about this insight is that it is not enough just to teach the methods, it is important to be aware of ‘in what ground the methods will grow’. If the teachers just continued their teaching and sort of assimilated the open problem concept, the teaching with open problems would be less beneficial for them and their pupils, than if they tried to accommodate the idea in their teaching system by reflecting on what kind of basis they had themselves to work with open problem-solving; and from this point of conscious awareness decide what kind of competencies it is necessary to work with to improve in mathematics teaching. What the data shows us is that the ‘teachers’’ skills in working with the open math tasks on the course were very similar to the pupils' skills. To only use the textbook or let the written exam tasks serve as guidelines would not only influence the pupils' but also the teachers' mathematical skills.

In the redesigned courses, I wanted to find ways to teach that addressed some of those problems. It meant that I did not only want to teach the participants methods, I wanted to create a stimulating and safe environment where we could reveal to them some of their teaching habits. I wanted them become aware of their own tacit habits in their mathematical communication, and then give them tools for how to improve or change these habits. In that work, I concentrated on a format that could give them knowledge about themselves when they worked with mathematics, more than I developed the content for open problems. I still asked them how they understood the concept 'open problems', and some of them still answered that 'the more open the better', and I still listened to them and explained what happens, when the task is without direction. During the courses, they tried these ideas in practice and we reflected on the results; maybe I became more critical to the extremely open question such as 'what mathematics is in this bottle?', because I saw how it worked in school classrooms. Yet, I no longer believe in just showing the teachers what 'good tasks' are; they have to feel on their own what the difficulties are. How do their experiences, their mathematical skills, their habits, and their children – how do all these topics influence the way they can work with such open problems? I admit that I was much more critical towards the tasks the teachers produced in the later courses than I was before this research study, and I used more time to ask questions and discuss the tasks with them. My study taught me that I can give the teachers any number of good tasks and talk as much as I like; they will still use the tasks exactly as their tacit beliefs, habits, routines or school culture direct them to.

My research has shown that to teach the teachers to work effectively with open problems I had first to find out how the teachers could be made aware of their own habits and beliefs, and from that knowledge give them tools to work with open problem-solving. In the study, I created the guiding principles, but for the teaching I made a long list of questions they could work with and try out to see if any of these questions could be developed into powerful questions, I taught the teachers modelling processes so that they had a strategy to use with problems from the 'real world', I provided countless pieces of advice for open problems, but the main thing was the activities to promote the teachers' awareness of their own strengths and weaknesses and how to present tools for their own teaching to them. My educational advice is that work with open problem-solving in mathematics requires knowledge for how these methods fit into the teachers' beliefs and other teaching habits, and if they do not fit well, it is maybe not the open tasks they need to change.

I will repeat that I have no intention of judging the teachers' behaviour or way of teaching; in a way I see the teachers as victims just like the pupils, who are sometimes confused rather than helped, when they work with open problems in mathematics. I met teachers on the courses who were 'hungry' for learning how to facilitate a good and effective practice, but I realised that we had different views of what effective teaching was. In a rough generalisation, the teachers considered the teaching effective if all the pupils worked with 'something/anything' that had mathematical relevance, and they were more occupied with the pupils' social relations than with the mathematics they learned, whereas I feel that the mathematical concepts should be challenged through the tasks, the work forms and the communication. One of the problems I became aware of was the explicit insufficiency of professional knowledge, including mathematical and mathematics educational competencies. This calls attention to that the educational decision-makers have not designed an education that equips mathematics teachers with enough competencies to teach mathematics in a satisfactory way when they leave the pre-service training. The mathematics teachers have a big task to do in their professional capacity, but they do not receive the necessary training to satisfy these requirements. One first step is always to be aware of the problems, not just among the teachers, but even more so among the decision-makers. In this study, I tried to demonstrate some of the problems to the teachers, but that is not enough. The problems I became aware of in this study have bigger implications than I can solve with an in-service course. Further research is called for, as is bold political action.

Reference List

Alrø H., Skovsmose O. (2002): *Dialogue and learning in Mathematics Education*. Dordrecht: Kluwer Academic publisher.

Andersen T. (2005): *Reflekterende processer*. København: Dansk psykologisk forlag.

Artigue M. (1994): Didactical engineering as a framework for the conception of teaching products. Biehler R. and others.: *Didactics of mathematics as a scientific discipline*. 27-39. Dordrecht: Kluwer Academic Publisher.

Atkinson P., Hammersley M. (1998): Ethnography and Participant Observation. Denzin N.K. and Lincoln Y.S.: *Strategies of Qualitative Inquiry*. Thousand Oaks: SAGE Publications.

Ball D.L. (1988): Unlearning to Teach Mathematics. In *For the learning of mathematics 8*. 40-48. Montreal, Quebec, Canada: FLM Publishing Association.

Ball D.L., Bass H. (2000): Interweaving Content and Pedagogy in Teaching and Learning to Teach: Knowing and Using Mathematics. Boaler J.e.: *Multiple Perspectives on Mathematics Teaching and Learning*. 83-104. London: Ablex publishing.

Ball D.L., Bass H. (2003): Making Mathematics Reasonable in School. Kilpatrick J. and others.: *A Research Companion to Principles and Standards for School Mathematics*. 27-44. USA: National Council of Teachers of Mathematics.

Barak S., Squire K. (2004): Design-based Research: Putting a Stake in the Ground. In *The journal of the Learning Sciences, 13(1)*. 1-14. Lawrence Erlbaum Associates, Inc.

Bassey M. (1995): *Creating Education Through research*. Edinburgh: British Educational Research Association.

Bateson G. (1972): *Steps to an ecology of mind*. San Francisco: Chandler.

Becker J.P., Shimada S. (1997): *The open-ended Approach: A new Proposal for Teaching Mathematics*. Reston, Virginia: National council of teachers of mathematics.

Becker J.P., Selter C. (1996): Elementary School Practices. Bishop A.J. and others.: *International Handbook of Mathematics Education*. [14], 511-564. Dordrecht, The Netherlands: Kluwer Academic Publisher.

Black P., Harrison Christine, Lee C., Marshall B., Wiliam D. (2002): *Working inside the black box*. King's College London: Department of Education & Professional Studies.

Black P., Wiliam D. (1998): *Inside the black box*. King's College London: School of Education.

Blomhøj M., Jensen T.H. (2002): *Developing mathematical modelling competence: Conceptual clarification and education planning*. Roskilde: Centre for research in learning Mathematics.

Blum W., Niss M. (1991): Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. In *Educational Studies in Mathematics, volume 22*. 22, 37-68. Dordrecht, the Netherlands: Kluwer Academic Publisher.

Brookes M. (1988): *Tegn med børn*. København: Nyt Nordisk Forlag.

Brousseau G. (1997): *Theory of Didactical Situations in Mathematics*. The Netherlands: Kluwer Academic Publishers.

Brown S.I., Walther M.I. (1993): *Problem posing: Reflections and Application*. Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.

Brown S.I., Walther M.I. (1990): *The Art of Problem Posing*. Hillsdale, New Jersey: Lawrence Erlbaum Associates Publishers.

Brown T. (1997): *Mathematics Education and Language*. Dordrecht: Kluwer Academic Publishers.

Bruner J. (1998): *Uddannelseskulturen*. København: Munksgaard.

Chevallard Y. (1985): *La transposition didactique*. Grenoble: La pensé Sauvage.

Chevallard Y. (1999): Didactictique? Is it a plaisanterie? You must be joking! A critical comment on terminology. In *Instructional Science*. 27, 5-7. The Netherlands: Kluwer Academic Publishers.

Christiansen B. (1990): *Interactive aspects of mathematics teaching and learning*. [Paper presented at the 4th SCTP Conference, Brakel, Germany, September 16-21]. Copenhagen: Royal Danish School of Educational Studies.

Christiansen I.M., Nielsen L., Skovsmose O. (1997): Ny mening til begrebet refleksion i matematikundervisningen. Jacobsen J.C.: *Refleksive læreprocesser*. 173-191. København: Forlaget politisk revy.

Clarke D. (1996): Chapter 9: Assesment. Bishop A.J.et.al.e.: *International Handbook of Mathematical Education*. 327-370. Netherlands: Kluwer Academic Publishers.

Clarke D., Clarke B. (2005): *Effective Professional Development for Teachers of Mathematics: Key Principles from Research and a Program Embodying These Principles*. Paper presented at ICMI Study 15, Águas de Lindóia, Brasil.

Cobb P., C.J., DiSessa A., Lehrer R., Schauble L. (2003): *Design experiments in educational research*. Educational researcher, 32(1).

Cooney t.J., Krainer K. (1996): Inservice Mathematics Teacher Education: The importance of Listening. Bishop A.J. and others.: *International Handbook of Mathematics Education*. 1155-1185. Dordrecht, the Netherlands: Kluwer Academic Publisher.

Covey S.R. (1992): *The Seven Habits of Highly Effective People: Restoring the Character Ethic*. London: Simon & Schuster.

Cummins D.D. (2000): How the social environment shaped the evolution of mind. In*: Synthese*. 3-28. The Netherlands: Kluwer Academic Publisher.

CVU København og Nordsjælland, CVU Storkøbenhavn, CVU Øst. (2002): *Efter- og videreuddannelse*. København: CVU.

Dahler-Larsen P. (1997): Orfeus blik - om undervisningsevaluering. Mai A.-M. and others.: *Imod en ny videnskabelig dannelse*. 215-237. Odense: Odense Universitetsforlag.

Dale E.L. (1989): *Pedagogisk profesjonalitet*. Norge: Gyldendal Norsk Forlag A/S.

Danmarks Evalueringsinstitut. (2002): *Folkeskolens afgangsprøver - Prøvernes betydning og sammenhæng med undervisningen*. København: Danmarks Evalueringsinstitut.

Danmarks Evalueringsinstitut. (2006): *Matematik på grundskolens mellemtrin*. København: Danmarks Evalueringsinstitut.

DiSessa A., Cobb P. (2004): Ontological Innovation and the role of theory in Design Experiments. 77-103. In *The journal of Learning Sciences* 13(1): Lawrence Erlbaum Associates, Inc.

Dreyfus H.&.S. (1999): Mesterlære og eksperters læring. Nielsen K. and Kvale S.: *Mesterlære*. 54-75. København: Hans Reitzels Forlag.

Dysthe O. (1997): *Det flerstemmige klasserum*. Århus DK: KLIM.

Dysthe O. (2003): *Dialog, Samspil og Læring*. Århus: Klim.

Ejersbo L.R., Nyholm A. (2004): *Didaktik i praksis. Matematik og natur/teknik i 5. og 6. klasse*. København: Learning Lab Denmark.

Ejersbo L.R. (2001): *Matematik og projektarbejde - hvorfor ikke?* København: DPU.

Ejersbo L.R., Andersen M.W. (1997): *Ideer til den mundtlige prøve i matematik. Åbne opgaver*. København: Danmarks Lærerhøjskole.

Ejersbo L.R., Leron U. (2005): *The didactical transposition of didactical ideas: The case of the virtual monologue.* Paper presented at the Fourth Congress of the European Society for Research in Mathematics Education, Sant Feliu de Guíxols, Spain.

Ejersbo L.R., Michelsen C. (2005): Hvor er facitlisten? Tanker om efteruddannelse af undervisere. Andresen M. and Thorslund J.: *Lærere i bevægelse*. 175-186. København: Roskilde Universitetsforlag & Learning Lab Denmark.

Ejersbo L.R., Mogensen A. (2001): *Faktor. Lærerens Håndbog i Syvende*. København: Malling Beck.

Ejersbo L.R., Engelhardt R., Frølunde L., Hanghøj T., Magnussen R., Misfeldt M. (2007): Balancing product design and theoretical insight. In Kelly A. (ed): *The Handbook of Design Research in Mathematics, Science and Technology.* Mahwah, NJ: Lawrence Erlbaum

Evans J.St.B.T. (2003): *In two minds: dual-process accounts of reasoning.* 7 No. 10, 454-459.

Forsythe D.E. (1998): Using ethnography to investigate life scientists' information needs. In *Bull Med Libr Assoc 86(3)*. 402-408. Stanford, California: Bull Med Libr Assoc 86(3).

Freudenthal H. (1991): *Revisiting mathematics education*. Dordrecht, the Netherlands: Kluwer Academic Publisher.

Gattegno C. (1970): *What We Owe Children*. New York: Outerbridge & Dienstfrey.

Glaserfeld E.v. (1995): *Radical constructivism. A way of knowledge and learning.* London: The Falmer Press.

Glesne C. (1999): *Becoming Qualitative Researchers, An introduction, Second edition.* USA: Pearson Education, Inc.

Goldenberg P. (1993): *On Building Curriculum Materials That foster Problem Posing*. Brown S. and Walther M.I.: *Problem Posing, reflections and applications.* 31-39. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Gravemeijer K., Cobb P., Bowers J., Whitenack J. (2000): Symbolizing, Modeling, and Instructional Design. Cobb P. and others.: *Symbolizing and communicating in mathematics classroom. Perspective on discourse, tools, and instructional design.* 225-273. London: Lawrence Erlbaum Associates, Publishers.

Grønsved W., J.Siemsen, B.Jensen, N.Egelund, P.Schultz Jørgensen. (1998): *Ønsker til fremtidige efteruddannelser samt vurdering af eksisterende.* København: Danmarks Lærerhøjskole.

Handal G., Lauvås P. (2002): *På egne vilkår - en strategi for vejledning med lærere.* Århus: Klim.

Hansen H.C.et.al. (2006): *Matematikundervisningen i Danmark i 1900-tallet.* København: In print.

Hansen T. (2001): *Fødevare kvalitet - et forbrugersperspektiv.* København: Jurist- og Økonomiforbundets Forlag.

Henriksen T.D. (2006): *Educational role-play: moving beyond entertainment. Seeking to please or aiming for the stars.* Tampere, Finland: Conference paper presented "On Playing Roles"-seminar.

Hohr H., Pedersen k. (1996): *Perspektiver på æstetiske læreprocesser.* København: Dansklærerforeningens forlag.

Hviid P. (2003): Til forholdet mellem undervisning og læring. Belyst gennem universitetspædagogisk praksis. In *Kognition & Pædagogik, Nr. 50, 13. årgang.* 51-70.

Jarvis P. (1999): *The practitioner-researcher. Developing Theory from Practice.* San Francisco: Jossey-Bass.

Jaworski B. (2004): Insiders and outsiders in mathematics teaching development: the design and study of classroom activity. edited by Olwen McNamara and Richard Barwell.: *Research in mathematics education volume 6.* 3-22. England: British Society for Research into Learning Mathematics.

Jensen H.N. (1996): *Fagets formål, læseplaner og evaluering. Unge Pædagoger, 5/1996.*

Jordan B., Henderson A. (1995): Interaction Analysis: Foundations and Practice. In *The journal of the Learning Sciences 4(1).* 39-103. Lawrence Erlbaum Associates.

Jørgensen T.H., Madsen U.A., Moos L., Thomassen J. (1992): *Visioner og paradokser.* København: Unge Pædagoger.

Karpatschof B. (2006): *Vejen ad hvilken (Metahodos).* København: In print.

Kelly A.E. (2005): Quality Criteria for Design Research: Evidence and Commitments. Edited by van den Akker J. and others.: *Educational Design Research.*

Kelly A.E.ed. (2007): *The Handbook of Design Research in Mathematics, Science and Technology Education.* Mahwah, NJ: Lawrence Erlbaum.

Kelly A.E. (2003): Research as Design. In *Educational Researcher Vol 32, No. 1.Theme Issue: The Role of Design in Educational Research*. 3-4.

Korsgaard O. (1999): *Kundskabskapløbet. Uddannelse i videnssamfundet*. Copenhagen: Gyldendal.

Krainer K. (2001): Teachers' Growth is MoreThan the Growt of Indivual Teachers: The Case of Gisela. Lin F.-L. and Cooney T.J.: *Making Sense of Mathematics Teacher Education*. [13], 271-294. Dordrecht: Kluwer Academic publishers.

Kvale S. (2001): *InterView. En introduktion til det kvalitative forskningsinterview*. København: Hans Reitzels Forlag.

Lampert M. (1990): When the Problem Is Not the Question and the Solution Is Not the Answer: Mathematical Knowing and Teaching. In *American Educational Research Journal, Vol 27, No. 1*. 29-63.

Lampert M. (2001): *Problems of Teaching*. New Haven & London: Yale University Press.

Larsen S. (1998): *Den ultimative formel - for effektive læreprocesser*. København: Eget Forlag.

Laursen P.F. (1997): Refleksivitet i didaktikken. Jacobsen J.C.: *Refleksive læreprocesser*. 60-77. København: Forlaget Politisk Revy.

Laursen P.F. (2004): *Den autentiske lærer*. København: Gyldendals Lærerbibliotek.

Lave J., Wenger E. (1991): *Situated learning:Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.

Leron U., Hazzan O. (1997): The world according to Johnny: A coping Perspective in Mathematics Education. In *Educational Studies in Mathematics*. 32, 265-292. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Lester F.K. (1998): Does the research reported in mathematics education journals have any relevance for practicing teachers? Pursuit of Practical Wisdom, In *Mathematics Education Research*. No 3-4, 71-82.

Linell P. (1998): *Approaching Dialogue. Talk, interaction and contexts in dialogical perspectives*. Amsterdam/Philadelphia: John Benjamins Publishing Company.

Luhmann N. (2000): *Sociale systemer*. København: Hans Reitzels Forlag.

Ma L. (1999): *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associated, Publishers.

Mason J. (1998): Enabling Teachers to be Real Teachers: Necessary Levels of Awareness and Structure of Attention. In *Journal of Mathematics Teacher Education 1*. 243-267. The Netherlands: Kluwer Academic Publishers.

Mellin-Olsen S. (1996): *Oppgavediskursen i matematik*. 6. www.caspar.no/tangenten/ta96/oppgavediskurs.html: Caspar Forlag.

Mezirow J. (1990): How Critical Reflection Triggers Transformative Learning. Jack Mezirow (ed.).: *Fostering Critical Reflection in Adulthood - a Guide to Transformative and Emancipatory Learning*. 1-20. San Francisco: Jossey-Bass Publisher.

Ministeriet for Videnskab. (2006): *Kvalitet i undervisningen*. København: Videnskabsministeriet.

Ministry of Education. (2002): *Facts and Figures. Education Indicators Denmark - June 2002*. Copenhagen: Ministry of Education, National Authority for International Institional Affairs, Statistics and Information Division.

Morgan C. (1997): The institutionalisation of open-ended investiagtion: some lessons from the U.K. experience. Pehkonen E.: *Use of open-ended problems in mathematical classroom*. 49-62. Helsinki: Department of Teacher Education, University of Helsinki.

Niss M. (1999): Aspects of the Nature and State of Research in Mathematics Education. In *Educational Studies in Mathematics 40*. 1-24.

Niss M. (2003): Mathematical competencies and the learning of mathematics: The Danish KOM project. Gagatsis A. and Papastavridis S.: *3rd Mediterranean Conference on Mathematical Education 3-5 January 2003*. 115-124. Athens: Hellenic Mathematical Society.

Niss M. (2006): *The concept and role of theory in mathematics education*. Roskilde University, Denmark: IMFUFA.

Niss M., Jensen T.H. (2002): *Kompetencer og matematiklæring*. København: Undervisningsministeriet.

Nordenbo S.E. (1997): *Fagdidaktik*. København: Gyldendal.

Palm T. (2002): *The realism of mathematical school task. Features and consequences*. Umeå University: Department of Mathematics.

Palm T., Burman L. (2004): Reality in mathematics assessment: An analysis of task-reality concordance in Finnish and Swedish national assessments. In *Nordisk matematikkdidaktikk, 9, no 3, October 2004*. 9, no 3, October 2004, 1-33. Göteborgs Universitet: NCM/NOMAD.

Patton M.Q. (2001): *Qualitative Research & Evaluation Methods*. 3. Thousand Oaks, London, New Delhi: Sage Publications.

Pehkonen E. (1997): *Use of open-ended problems in mathematics classroom*. Helsinki: Department of Teacher Education University og Helsinki.

Pehkonen E., Törner G. (2004): *Methodological considerations on investigating teachers' beliefs of mathematics and its teaching*. *Nordisk matematikkdidaktikk*. 9, No 1, January, 21-50. Göteborg, Sweden.

Piaget J. (1973): *Intelligensens psykologi*. København: Hans Reitzels Forlag.

Pinar W.F., William M.Reynolds, Patrick Slattery, Peter M.Taubman. (1995): *Understanding Curriculum*. New York: National Academy Press.

Pirie S.I.B., Borgen K., Manu S.S., Jenner D., Thom J., Martin L.C. (2001): Theory, video and mathematical understand-ing: an examination of what different theoretical perspectives can offer. R.Speiser C.A.M.&.C.N.W.E.: *Proceedings of the twenty-third annual meeting of the*

North American chapter of the international group for the psychology of education. 343-380. Columbus: ERIC Clearinghouse of Science, Mathematics, and Environmental Education.

Powell A.B., Franscisco J.M., Mahler C.A. (2003): An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. In *Journal of Mathematical Behavior 22*. 405-435. Elsevier Inc.

Rasmussen J. (2004): *Undervisning i det refleksivt moderne*. København: Hans Reitzels Forlag.

Rigbolt P. (2000): Den tre-benede strategi for efteruddannelse. Christensen C.U.&.G.S.: *Dialog om efteruddannelse*. 1-11. Århus: Kroghs forlag.

Schmidt L.H. (2000): Forskningstilknytning betyder tilknytning til forskning. *DPU Intranet* p.1-7. København: DPU.

Schoenfeld A. (1992): Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. Grouws (ed.).: *Handbook of Research on Mathematis Teaching and Learning*. New York: Macmillan Publishing Company.

Schön D.A. (1983): *The Reflective Practitioner*. USA: Basic Books, Inc.

Sfard A., Kieran C. (2001): Preparing Teachers for Handling Students' Mathematical Communication: Gathering Knowledge and Building Tools. Lin F.-L. and Cooney t.J.: *Making Sense og Mathematics Teacher Education*. 185-205. Dordrecht: Kluwer Academic Publishers.

Shulman L.S. (1987): Knowledge and teaching: Foundation of the new reform. *Harvard Educational Review, 57/1*. 1-22.

Simon M.A. (2000): Constructivism, Mathematics Teacher Education, and researcher in Mathematics Teacher Development. Steffe L.P. and Thompson P.W.: *Radical Constructivism in Action: Building on the Pioneering Work of Ernst Von Glasersfeld*. [14], 213-230. London: Taylor & Francis.

Skott J. (2000): *The Images and pratice of Mathematics Teachers*. [215]. København: The Royal Danisk School of Educational studies.

Skott J. (2004): The forced autonomy of mathematics teachers. In *Educational Studies in Mathematics 55*. 227-257. The Netherlands: Kluwer Academic Publishers.

Skovsmose O. (1994): *Towards a Philosophy of Critical Mathematics Education*. The Netherlands: Kluwer Academic Publisher.

Skovsmose O. (2000): *Landscapes of investigation*. [24]. Centre for Research in Learning Mathematics.

Stanovich Keith E., West R.F. (2003): Evolutionary Versus Instrumental Goals: How Evolutionary Psychology Misconceives Human Rationality. Over D.E.ed.: *Evolution and the psychology of thinking: The debate.* 171-230. Psychological Press.

Steinbring H. (1998): From "Stoffdidaktik" to Social Interactionism: An Evolution af Approaches to the Study of Language and Communication in German Mathematics Education Research. Steinbring H. and others.: *Language and communication in the classroom*. 102-119.

Steinbring H. (2005): *The construction of New Mathematical Knowledge in Classroom Interaction*. USA: Springer.

Steinbring H. (2006): What makes a sign a mathematical sign? - and epistemological perspective on mathematical interaction. In *Educational Studies in Mathematics*. 61/1, 133-162.

Stiegler J.W., Hiebert J. (1999): *The Teaching Gap*. New York: The Free Press.

Stiegler J.W., Hiebert J. (2004): At udveckla matematikundervisningen. *Nämnaren 31(1)*. 31 (1), 38-43. Göteborg: Göteborgs Universitet.

Strauss A., Corbin J. (1998): *Basics of Qualitative Research*. USA: Sage Publications, Inc.

Strauss S. (2001): Folk Psychology, Folk Pedagogy, and Their relations to Subject-Matter Knowledge. Torff B. and Sternberg R.J.: *Understanding and Teaching the Intuitive Mind*. 217-242. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Sullivan P., Clarke D. (1991): *Communication in the Classroom: The Importance of Good Questioning*. Victoria, Australia: Deakon University.

Thompson A.G. (1992): Teachers' Beliefs and Conceptions: A synthesis of the Research. Grouws (ed.).: *Handbook of Research on Mathematical Teaching and Learning*. New York: Macmillan Publishing Company.

Thyssen O. (1997): *Værdiledelse*. København: Nordisk forlag A.S.

Undervisningsministeriet. (2001): *Prøver, Evaluering, Undervisning. Matematik, Fysik/Kemi 2001. Uddanelsesstyrelsens håndbogsserie nr. 10-2001, Grundskolen*. København: Undervisningsministeriets forlag.

Undervisningsministeriet U.F. (1998): *Prøverne i matematik. Bekendtgørelse og vejledning*. København: Undervisningsministeriets Forlag.

van Oers B. (2000): The Appropriation of Mathematical Symbols: A Psychosemiotic Appraoch to Mathematics Learning. Cobb P. and others.: *Symbolizing and communicating in mathematica classrooms. Perspectives on discourse, tools, and instructional design*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.

Voigt J. (1996): Negotiation of Mathematical Meaning in Classroom Processes: Social Interaction and Learning Mathematics. Steffe L.P. and others.: *Theories of Mathematical Learning*. 21-50. New Jersey: Lawrence Erlbaum Associates, Publisher.

Wahlgren B., Høyrup S., Pedersen K., Rattleff P. (2002): *Refleksion og læring. Kompetenceudvikling i arbejdslivet*. København: Samfundslitteratur.

Watson A., Mason J. (1998): *Questions and Prompts for Mathematical Thinking*. Derby: ATM.

Wedege T., Skott J. (2006): *Changing views and practices*. Trondheim: Norwegian Center for Mathematics Education.

Wells G. (1999): *Dialogic Inquiry. Towards a Sociocultural Practice and Theory of Education*. USA: Cambridge University Press.

Wenger E. (1998): *Communities of Practice. Learning, Meaning, and Identity*. Cambridge: cambridge University Press.

William D. (1994): Assessing authentic tasks: alternatives to mark-schemes. In *Nordic Studies in Mathematics Education*. 2 (1), 48-68.

Winsløw C. (2006): *Didaktiske elementer - en indføring i naturfagenes og matematikkens didaktik.* København: Biofolia.

Wistedt I. (2001): Rum för samtal. Grevholm B.: *Matematikdidaktik - ett nordiskt perspektiv*. 219-229. Lund, Sweden: Studentlitteratur.

Wittmann E.Ch. (1998): Mathematics Education as a 'Design Research'. Sierpinska A. and Kilpatrick J.: *Mathematics Education as a Research Domain: A search for Identity*. 87-103. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Wong E.D. (1995): Challenges Confronting the Researcher/Teacher: Conflicts of Purpose and Conduct. In *Educational Researcher, Vol. 24, No.3*. 22-28.

Wood T. (2006): Teacher education does not exist. In *Journal for Research in Mathematics Education*. 9, 1-3.

Wood T., Berry B. (2003): Editorial: What does "Design Research" offer Mathematics Teacher Education. In *Journal of Mathematics Teacher Education 6*. 195-199. Dordrecht: Kluwer Academic Publisher.

Yackel E., Cobb P. (1996): Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*. 27, No.4, 458-477.

Appendices

Appendix I: Questionnaire 1

Prøveforberedende kursus
i matematik
DPU august 2002

Midtvejsevaluering

Har du haft en god uge?

Nej ______ (Ja)

Var der noget du manglede?

Ikke noget, jeg kan "se" på nuværende tidspunkt

Noget der var for meget af?

Brug af computere til den mundtlige prøve.

Noget der var særlig godt?

Jeg synes det har været en fin uge helt igennem.

Spontane kommentarer efter en uges arbejde: (de styrede er på de næste sider)

En rigtig god og lærerig uge.
Jeg hentede ~~mig~~ mange gode ideer og masser af inspiration.
Mere af det!

Navn:

Prøveforberedende kursus
i matematik

DPU august 2002 - Midtvejsevaluering

Giv de enkelte elementer point fra 1-10. Skriv også hvad der var godt og hvorfor, samt hvad der kunne gøres bedre, hvordan.

§ Hvad siger loven?	Gav lidt mere overblik.
'Det gode prøve-spørgsmål'	Vedkommende. Gode oplæg og en diskussion, der "klarede" nogle problemer.
Allan Malmberg: EDB og de mundtlige prøver.	Spild af tid. Man kunne nøjes med at løse IT-(lære)mål(?) i matematik
Produktion af prøveoplæg	Fint
Prøve-eksamen	Gav nye erfaringer. Sjovt og noget man blev klogere af.
Inspiration til egne projekter.	Inspirerende. Det blev lettere at komme i gang.
Klodser og de store klasser samt mindre input	Fint - inspirerende
Produktion af egne oplæg	Meget tilfredsstillende at komme i gang.
Afslutning	

Appendix II: Questionnaire 2

CVU København & Nordsjælland
Center for Videregående Uddannelser

Uddannelsessekretariatet
Ferrodsgade 23 (?)
2200 København N
TLF. 35865510 FAX. 35865509
cvukbh@cvukh.dk
WWW.CVUKBH.DK

SLUTEVALUERINGSSKEMA – *deltagere*

Kære deltager.

CVU København & Nordsjælland satser på en stadig udvikling af kvaliteten i den undervisning, vi tilbyder. Det er derfor væsentligt for os at få en slutevaluering om kursusforløbet til brug for fremtidig kursusplanlægning og kursusudvikling. De bedes derfor udfylde omstående evalueringssider.

Med venlig hilsen

Lene Haderup
Uddannelsesleder

Kursustitel: Prøveforberedende arbejde i matematik

Kursusnummer: 80 - 2002 - 0114 - 0

Sted/lokale: EDB-lokaler (161 a & d) DPU

Underviser/koordinator: Lisser Rye Ejersbo m.fl.

Kursusstart- og slutdato: 26.08.02 – 15.11.02

Antal timer i alt: 48

---

1. Levede kurset (mål, indhold og form), i forhold til dine forventninger, op til kursusopslag og kursusprogram?

Slet ikke..X...Fuldt ud

På hvilken måde?
Meget inspirerende
god organisering og struktur

2. Havde du mulighed for løbende medindflydelse på kursusforløbet?

Slet ikke..X Fuldt ud

På hvilken måde?
De spørgsmål og frustrationer blev taget meget alvorligt

3. Bidrog du til, at kursusforløbet levede op til det planlagte program (mål, indhold og form)?

Slet ikke...X...............Fuldt ud

På hvilken måde?
Der var en del hjemmearbejde hvor jeg selv kunne være mere aktiv.

4. Har kurset medført/vil kurset medføre ændringer i dit daglige arbejde?

Slet ikke..Fuldt ud X

På hvilken måde? Jeg vil [illegible] mig til [illegible] til min praksis efter dette kursus. Godt og inspirerende for kursisterne som for [illegible]

5. Har kurset medført/vil kurset medføre ændringer på din skole?

Slet ikke...✓..........Fuldt ud

På hvilken måde? Så meget vi kan få til det!

6. Var de fysiske rammer (lokaler/hjælpemidler) tilfredsstillende?

På hvilken måde? Ja. Gode gruppelokaler o.l.

7. Hvilke elementer i kursusforløbet var særlig værdifulde for dig?

Samtale med kolleger om mundtlig prøve

8. Har du idéer til fremtidige kurser?

Samme emne – men mere vægt på udarbejdelse af prøveprogram.

9. Bemærkninger i øvrigt?

Navn:..

Skole/institution:..

Appendix III: Cola Task

Cola !

Cola er i dag en væsentlig ingrediens i de fleste unges liv. "Cola" findes i mange forskellig forklædninger, dvs. det laves af mange forskellige bryggerier, findes i mange forskellige slags emballage og til mange forskellige priser.

Det skal vi arbejde med på forskellige måder, og ud fra forskellige problemstillinger:

Fælles for alle:

1) Elevforbrug:
- Lav spørgeskema
- Lav statistik over de indsamlede oplysninger
- Lav et diagram over resultatet

2) Emballagen:
- Perspektivtegning – dåse, kasse,
- Målestoksforhold – tegning af kasse i målestoksforhold med huller til flasker
- Design af egen dåse til sodavand – lav selv logo
- Affald – udfra undersøgelsen af klassens cola-forbrug beregnes den samlede affaldsmængde i kg
- Kvantitativ beskrivelse – geometri, farve, størrelsesforhold,

3) Priser:
- Beskriv og vurdér mindst 4 forskellige colatyper med hensyn til pris og kvalitet
- Diskuter hvad der bestemmer prisen på en cola
- Er der nogle gode tilbud i denne uge på cola?

Gruppearbejde:

FEST

Forestil dig at klassen skal holde afslutningsfest for en pige, som flytter til en anden skole.

Forudsætninger:
Klassen får 35 kr. pr. elev til at holde festen for.
Af dette beløb bruges ca. 22 kr. til mad, chips, slik m.m.
Der drikkes kun cola til festen.

Brug jeres viden om cola til at planlægge indkøbene til festen.
- Lav et budget

Planlæg festen, den skal holdes i jeres klasseværelse :
- bordopstilling,
- bordplan (hvem sidder hvor),
- balkort
- dansegulv
- lav en skitse i målestoksforhold
-
-

FEST

Forestil dig at klassen skal holde afslutningsfest for en pige, som flytter til en anden skole.

Forudsætninger:
Klassen får 35 kr. pr. elev til at holde festen for.
Af dette beløb bruges ca. 22 kr. til mad, chips, slik m.m.
Der drikkes kun cola til festen.

Brug jeres viden om cola til at planlægge indkøbene til festen.
- Lav et budget

Planlæg festen, den skal holdes i jeres klasseværelse :
- bordopstilling,
- bordplan (hvem sidder hvor),
- balkort
- dansegulv
- lav en skitse i målestoksforhold
-
-

KIOSK

Din onkel har en kiosk i Istedgade. Med jævne mellemrum kører han til Tyskland med en anhænger på sin bil, for at købe dåse-colaer til at sælge i butikken. Han sælger dem for 5 kr. stykket.

Han synes ikke at salget i kiosken giver nok overskud, og du vil gerne hjælpe ham med at regne lidt på hans udgifter i forhold til salgsprisen – skal den evt. hæves lidt, eller skal han måske indkøbe/sælge en anden type cola end dem på dåse?

Tag beslutninger og nedskriv dine forudsætninger:
- Hvilken bil har din onkel. egenvægt, km/liter benzin,
- hvilken anhænger: mål og kapacitet.
- hvilket kørekort: hvor mange kilo må han transportere

Undersøg:
- Tailene for dåse-colaer: Vægt mål, pris i Tyskland ....
- Hvad andre cola-typer koster i Tyskland
- Hvor langt er der til Tyskland (husk at beslutte hvilken vej din onkel vælger at tage til Tyskland)

Beregn:
- De samlede udgifter i forbindelse med at hente en anhængerfuld dåsecola i Tyskland
- hvad din onkel tjener på en dåsecola.

Vurder din onkels udgifter i forhold til hans salgspris. Hvad vil du råde ham til? ________

Målgruppe og tidsforbrug :

8.-10. Klasse. 6 timer på en studiedag eller 8 enkelt timer.

Målsætning:

At eleverne anvender matematik i praksis
Udvikle samarbejdsevne
At udvikle de unges miljøbevidsthed
At udfordre elevernes æstetiske sans
At udvikle elevernes prisbevidsthed.

Faglige Færdigheder:

Udforme spørgeskema
Bearbejdning af indsamlede data
Udforme diagrammer
Kunne vælge og anvende et passende målestoksforhold
Kunne anvende geometrisk terminologi om en bestemt rumlig form (flaske)
Anvende geometri til eget design
Udregne gennemsnit
Veje og måle
Regning med fremmed valuta
Kombinatorik
Udregne afstande

Organisation:

Undervisningen organiseres med et kort læreroplæg, hvorefter eleverne arbejder selvstændigt i grupper på 2-3 personer. Læreren virker som inspirator og sørger for praktiske ting.
Gruppernes arbejde fremlægges på plancher samt mundtligt.

Materialeliste:

Forskellige colaemballager- sodavandskasse- computer- papir- karton- lim- saks- farver- vægt- landkort / afstandstabeller- tilbudsaviser- lokalaviser- Silvan- Bauhaus- eller Harald Nyborg Katalog

Appendix IV: Tasks

Ligninger

Fortæl en historie over hvad de nedenstående regneudtryk kunne stå for:

$2x =$

$7 - x =$

$\frac{1,52 + 1,67 + 1,61 + 1,57 + x}{5} =$

$2x + 1 =$

$37x =$

$43,25x =$

x^2

$360 - 2x =$

$2x * 3,14 =$

Ligninger

Lav selv nogle ligninger udfra nedenstående tekster.

Politibetjenten havde skudt efter røveren 3 gange. Betjenten havde i alt 8 skud.

Han havde fyldt tanken helt op 40 liter, men nu gik han i stå 512 km stod der på speedometeret.

De fik tre bananer mere. Nu kunne alle fem endelig få syv hver.

Efter at havde været ude på hendes 9 km løbetur, kunne hun lige skimte den kvadratiske mark hun havde løbet rundt om 1 km borte.

Konstruerer nogle ligninger udfra hvert begreb.

Mobiltelefon.

Fritidsinteresse

Madlavning

Ligninger

En lærer fra Højdevangens Skole er med sin klasse på "bygge og anlæg" i en uge. Læreren laver en fantastisk trøje, der kan sælges ved kommende demonstration på Rådhuspladsen. Læreren vil bruge eleverne som billig arbejdskraft.
Stedet, bygge og anlæg skal have 100 kr. i timen for at stille lokaler til rådighed.
Trøjen koster 25 kr. i indkøb og sælges til 50 kr. to elever laver en trøje på 3 min. Læreren vil tjene 1000 kr.

Lav nogle ligninger over forskellige problemstillinger.

Kan læreren tjene 1000 kr.

Skal eleverne have penge for deres arbejde.

Resume: Re-design af et efteruddannelseskursus for matematiklærere

Problemstillingen

Nærværende afhandling drejer sig om efteruddannelse af matematiklærere i folkeskolen. Der bruges mange økonomiske ressourcer på efteruddannelse af forskellige lærerfaggrupper, men der vides meget lidt, hvilken effekt disse kurser har. Afhandlingen indeholder et forstudie og et hovedstudie. Forstudiet undersøgte udbyttet af et efteruddannelseskursus om brugen af åbne praktiske problemstillinger i matematikundervisningen, ved at se på lærernes egen efterfølgende undervisning. Hovedstudiet undersøgte, hvordan kursusdesignet kunne udvikles både teoretisk og praktisk på basis af viden fra forstudiet og gennem anvendelse af 'design research' som forskningsmetode. Design research er en metode, der ikke alene udvikler et artefakt (her efteruddannelseskurset) men også sideløbende udvikler teorier på den baggrund.
Forskningsspørgsmålet er som følger:
I hvor høj grad og på hvilken måde kan en meta-didaktisk transposition blive indarbejdet i en række gentagne kursers design, og hvilken effekt har den, undersøgt i forhold til deltagernes reaktioner på kurset.
Det pågældende efteruddannelseskursus drejede sig om, hvordan matematiklæreren kunne forberede sig til at afholde den mundtlige prøve i matematik, som Undervisningsministeriets bekendtgørelse nr. 639 af 21. juli 1995 ønskede den skulle se ud: En gruppeprøve med grupper i størrelse 1-3, med et oplæg, der bygger på en praktisk problemstilling, gerne af åben karakter. Kurset blev oprindeligt designet og gennemført af undertegnede i 1996, hvor der første gang deltog 100 lærere. Siden har jeg afholdt det samme kursus mindst en gang om året. Lærere gav overvejende udtryk for stor tilfredshed med deres deltagelse i kurset, og i 2002 fik jeg gennem et ph.d. stipendium mulighed for at undersøge, hvordan denne tilfredshed effektuerede sig i en efterfølgende klasseundervisning. Jeg har i dette studie været dels underviser på kurset og dels forsker i projektet, det har haft både sine svagheder og styrker. En af svaghederne består i at kunne skelne mellem de forskellige intentioner en underviser og forsker har til et projekt. Underviseren ønsker at påvirke sine studerende ved at bibringe dem ny viden og reagerer, hvis der ikke sker en forventet forandring. Forskeren derimod ønsker at undersøge en sag uden at påvirke den mere end højst nødvendig for derigennem at opnå forståelse for et fænomen. Denne modsætning måtte jeg håndtere flere gange i løbet af projektet.
Som basis for mine undersøgelser har jeg i afhandlingens første del beskrevet de love og bekendtgørelser, der knytter sig til det genstandsfelt, hvori den mundtlige prøve i matematik befinder sig. Løbende krav fra ministerielt hold stemmer ikke altid overens med de faktiske kompetencer, lærerne er uddannede til at håndtere. Efteruddannelseskurser er derfor ment som en hjælp til at overkomme disse diskrepanser. Det undersøgte kursus tilhører denne kategori.
Indholdet på kurset drejede sig om: 1) Hvordan man kan formulere åbne praktiske problemstillinger til benyttelse i både den afsluttende prøve og den daglige undervisning, og 2) Hvordan kommunikation i matematikundervisningen kan udvikles.
Praktiske problemstillinger med en åben karakter kræver definitioner af alle tre begreber: Problemstilling, praktisk og åben. Min definition på en problemstilling er inspireret af Blum og Niss (1991) og lyder således: For at kalde en tekst eller spørgsmål for en problemstilling, skal der være tale om noget, der er et problem for nogen. Det skal i denne kontekst yderligere være en matematisk problemstilling, hvilket vil sige at en løsning på problemstillingen skal kunne findes og kvalificeres gennem anvendelse af matematik.

Praktisk er ligeledes et flertydigt ord. Det kan enten betyde 'hands-on' eller være en beskrivelse af en praktisk situation udenfor matematikken. En situation udenfor matematikken kræver, at man gennem en matematisk modelleringsproces omdanner problemstillingen til et sæt data, der kan bearbejdes matematisk (Blomhøj og Jensen, 2002). Også dette kan foregå på flere forskellige måder. Hvis problemstillingen yderligere skal være autentisk, hvilket der lægges op til fra ministeriets side, må den opfylde nogle krav, som ministeriet dog ikke har udspecificeret nærmere. En opgaves autenticitet er et spørgsmål om graden af bevarelse af den virkelige situation, som opgaven er udsprunget af (Palm og Burman, 2004). Kriterier for autenticitet drejer sig om, hvorvidt den beskrevne situation er reel, om man har man de samme hjælpemidler til rådighed, som i den virkelig situation, om man ved, hvem der har brug for eventuelle løsninger og til hvad, og om oplysningerne i opgaven er autentiske.

Det åbne koncept udgør et tredje krav. Udtrykket 'åben opgave' bruges ofte i flæng blandt lærere, men med mange forskellige betydninger. Fra ministerielt hold udstikkes en opgaves åbenhed på 4 forskellige måder: I den indledende præsentation, i valg af arbejdsmetoder, i valg af fremgangsmåder eller i produktet, der kan være mere eller mindre fastlagt på forhånd. Der gives ingen forklaring på forskellen mellem arbejdsmetoder og fremgangsmåder, som begge præsenterer selve arbejdsprocessen. I Japan har åbne opgaver været anvendt siden 70'erne og her defineres en opgave som åben, hvis indgangen til opgaven er åben, hvis processen til svaret er åben eller hvis det endelig produkt er åbent (Becker og Selter, 1996). Til forskel tillader en lukket opgave kun en bestemt fremgangsmåde og et kendt facit. Disse beskrivelser drejer sig om opgavens formulering, men en matematikopgave, der er knyttet til en mundtlig kontekst, vil sjældent stå alene. Det betyder, at yderligere spørgsmål til en opgave kan dels åbne den mere, dels lukke den yderligere eller simpelthen gøre undersøgelser ligegyldige. En opgaves åbenhed bliver på den måde afhængig af, hvordan læreren stiller sine spørgsmål og hvilke svar, han eller hun forventer (Watson og Mason, 1998).

Forstudiet

Efteruddannelseskurset 2002 bestod af 1) en uges koncentreret undervisning på 30 timer og 2) 3 dages opfølgning. Opfølgningen indeholdt refleksioner over den undervisning, der var foregået i lærernes egne klasser i de mellemliggende to måneder. Kurset indeholdt mange aktiviteter, heriblandt fremstilling af gode prøveopgaver og planlægning af undervisningsforløb. I løbet af kurset blev forskellige kommunikationsformer præsenteret og afprøvet på forskellig vis. Karakteristisk for kurset var et højt aktivitetsniveau for deltagerne.

Empirien i forstudiet drejede sig om at undersøge effekten af kurset, undersøgt gennem interviews og videooptagede observationer i en efterfølgende klasseundervisning. Fortolkningen af disse observationer er baseret på kvalitative forskningsmetoder. Fire deltagende lærere blev fulgt. De udtrykte alle tilfredshed med kurset, og at de havde ladet sig påvirke til at afprøve nye metoder i deres undervisning.

Fortolkningen af observationerne er baseret på kvalitative forskningsmetoder, fx så jeg videoerne igennem adskillige gange for at finde karakteristiske ting for undervisningen med åbne opgaver, før jeg transskriberede optagelserne. Videoerne viste, hvor forskelligt arbejdet med åbne opgaver blev grebet an i klasserne, og jeg noterede mig, hvor besværlighederne lå. Min kategorisering af disse besværligheder er:

- de åbne spørgsmål drejer sig ikke om matematik, men snarere om at have en mening om problemer af mere samfundsmæssig karakter

- der anvendes mange floskler ('buzzwords') i forklaringer, som læreren ikke selv har tænkt igennem
- de praktiske problemstillinger er uautentiske; det handler blot om, at der laves en eller anden form for matematik
- læreren lytter ikke til eleverne, før han kommer med et svar eller en anvisning.

Observationer viste, at konsekvensen af disse besværligheder var, at læreren ikke var i stand til at vejlede eleverne, men måtte acceptere alting, eller at eleverne ikke blev udfordret, hvor det var mest hensigtsmæssigt. Overordnet er der tale om lærerens kommunikation og forventninger, når matematikundervisningen drejer sig om åbne opgaver.

Disse besværligheder kunne skyldes mange ting, men i dette studie besluttede jeg at undersøge, hvordan undervisningen foregik på kurset, når det drejede sig om netop disse problemstillinger. Hvordan jeg med andre ord selv underviste i kommunikation og refleksion over samme. Denne undersøgelse afslørede, at jeg faktisk slet ikke underviste i kommunikation. Aktiviteterne fik lov at tale deres eget sprog, og jeg overlod refleksionen til lærerne selv i form af gruppearbejde. Lærerne fik derfor ikke redskaber, der kunne hjælpe dem til at blive bevidste om egen kommunikation, ej heller fik de modeller til, hvordan det kunne gøres anderledes.

Konklusioner af forstudiet

I min bearbejdning af data undrede jeg mig over, hvorfor lærerne var så glade for kurset, når jeg nu oplevede så stor uoverensstemmelse mellem mine intentioner og det, jeg faktisk så. En forklaring kunne være, at undervisningen på kurset ikke var forstyrrende nok for lærernes allerede tillærte vaner, eller at de ikke blev præsenteret for alternativer til deres aktuelle undervisning. Det åbne koncept blev som konsekvens heraf blot omformet til at passe ind i allerede eksisterende vaner og gav ikke den fornyelse, jeg havde forventet eller håbet på. En af de interviewede lærere sagde: 'Hvordan skulle jeg vide det?', hvormed hun mente at kunne undervise anderledes. Denne sætning beskriver en kerne i lærernes læring: Lærerne underviser, som de har lært gennem en lang socialisering i egen skoletid og tiden på lærerseminariet. Hvis de deltagende lærere skal lære noget nyt, skal det gøres tydeligt, hvad de skal lære. At være lærer er at kunne påtage sig en professionel rolle, hvor både matematisk kommunikation og andre nødvendige kompetencer kan læres.

Hovedstudie

Hovedspørgsmålet i denne del af studiet blev, hvordan kurset kunne re-designes på basis af den viden, der var indhentet gennem forstudiets empiri. Begrebet en 'meta-didaktisk transposition' er inspireret af Chevallard's (1985) begreb 'didaktisk transposition'. En didaktisk transposition er den forandringsproces videnskabelig matematik undergår, når den skal præsenteres for en elevgruppe og derfor tilpasses gruppens opfattelseszone, samtidig med at den stadigvæk skal være en synlig repræsentation af den oprindelige sag. 'Meta' i mit begreb går på selve indholdet, der er i dette studie betyder matematikdidaktik. Jeg opfatter undervisning som et middel til formidling af viden om noget. Hvis vi opfatter midlet som redskab mellem sagen og den elevgruppe der skal opnå viden, skal redskabet passe til både sagen og gruppen, på samme måde som skruetrækkeren skal passe til både hånden og skruen. For at tilgodese denne metafor og min egen kritik af 2002-kurset udviklede jeg et sæt vejledende principper, som jeg mente kunne tilgodese nogle af de mangler i kursusundervisningen, som jeg havde konstateret. De tre vejledende principper (VP) er som følger:

1.
Ethvert undervisningsforløb på kurset er baseret på en kendt teoretisk ide eller ramme
2.
Teorien er omsat til en aktivitet for kursusdeltagerne. Aktiviteten overholder følgende regler:
   a. Undervisningselementerne praktiserer teorien og gør den således synlig
   b. Undervisningsmetoderne er mulige at overføre til anden praksis
   c. Aktiviteten synliggør deltagernes forforståelse eller relevant tavs viden
   d. Aktiviteten indeholder feedback mekanismer for relevant refleksion
3.
I planlægningen vælges teorien først, dernæst designes aktiviteten. I praksis gennemføres aktiviteten før teorien præsenteres.

Disse principper bygger på, at der skal være aktiviteter i undervisningen, men at disse aktiviteter skal være et udtryk for den teori, som skal præsenteres for deltagerne. Principperne bygger på, at aktiviteter kan åbne for en følelsesmæssig tilstand, men samtidig skal de også medvirke til at deltagernes kunnen og vaner i den pågældende kontekst bliver synlige. Det første skridt for en forandring bliver således gennem en bevidsthed om egne kompetencer og vaner. Teorien kommer sidst, hvor deltagerne får mulighed for at stå på 'skuldrene af giganterne' gennem et kendskab til deres teorier.
Den empiriske del af hovedstudiet bestod i at undersøge, hvordan denne nye kursusundervisning påvirkede lærerne, mens de var på kurset. Selve designet af kurset var det artefakt, jeg undersøgte, udviklede og teoretiserede over med hjælp fra videooptagede observationer af deltagernes reaktioner.
Gennem tre forskellige omgange af kurset, gennemført en gang om året, blev det synligt hvordan aktiviteterne forårsagede, at deltagernes vaner og kunnen kom til udtryk. Aktiviteterne indeholdt ofte små rollespil, hvor deltagerne på skift var observatører. Observatørernes iagttagelser blev senere rapporteret til både den lille gruppe og til hele gruppen. Iagttagelserne blev på den måde genstand for diskussioner, inden en teori om emnet blev præsenteret. Gennem disse øvelser blev lærernes egne vaner og kunnen ikke blot synlige for dem selv, men også for mig. Det blev bl.a. tydeligt at deltagerne, som alle var matematiklærere i overbygningen (7.- 10. klasse),

- selv havde problemer med at løse forholdsvis enkle åbne matematikopgaver,
- at gruppearbejde ikke fungerer, hvis man ikke har en fælles diskurs eller strategi,
- at lærere sætter aktiviteter i gang uden at fastlægge mål, fordi det er selve aktiviteten, der er ønskværdig snarere end indholdet, og
- at (matematisk) kommunikation ikke blev reflekteret i udpræget grad.

Konklusion af hovedstudiet

VP- undervisningen blev udviklet gennem forløbene og viste sig effektiv i den måde deltagernes egne vaner og kompetencer blev synlige samtidig med, at der blev givet mulighed for alternativer. Flere af disse alternativer var udformet som konkrete redskaber for en klasserumspraksis i form af analyseredskaber eller udformning af en række hensigtsmæssige spørgsmål.
De anvendte rollespil fik deltagernes praksis i spil, og udløste på den måde en vis 'afsløring' af lærerens handlinger. Det var hensigten med dem, men samtidig kunne det af deltagerne opleves som en ubehagelig udlevering. Det reagerede de meget forskelligt på. Deres reaktioner kan kategoriseres i to modsatte retninger, hvor forsvar var den ene pol, mens begejstring og åbenhed for at lære nyt gennem bevidsthed om egne 'fejl' var den anden. Jeg observerede reaktioner langs hele dette

spekter. Disse lærere blev ikke fulgt efter kurserne, så jeg kan ikke udtale mig om nogen langtidseffekt. Reaktionerne på kurset, som var studiets undersøgelsesobjekt, viste sig at være meget anderledes end i mine tidligere kurser. Min konklusion på denne meta-didaktiske transposition er, at den frembragte større bevidsthed om egen praksis hos deltagerne på kurset. Aktiviteterne afdækkede samtidig uventet en større afstand mellem deltagernes aktuelle kompetencer og de kompetencer, det kræver at gennemføre en hensigtsmæssig matematikundervisning med åbne opgaver.

Generelle konklusioner og spekulationer

Mine empiriske data afdækker forskellige lærerkompetencer af både didaktisk-pædagogisk art såvel som af faglig art. Jeg kan dog ikke pointere nok, at jeg ikke ønsker at hænge lærerne ud for eventuelle manglende kompetencer, men snarere opfatter jeg, at lærerne, såvel som eleverne, er ofre for den samme systemfejl. Lærere underviser nemlig, som de har lært.
Mine konklusioner på de indsamlede data er med andre ord, at de synliggjorte resultater viser en systemfejl, der drejer sig om, at lærere ikke lærer de kompetencer, der er nødvendige for at kunne håndtere de krav, der fra officielt hold stilles til matematiklæreren. Det er derfor et spørgsmål af politisk karakter at uddannelse skal rette sig mod de nødvendige kompetencer, som skal kunne fungere i en efterfølgende praksis. Udviklingen af de vejledende principper gjorde en positiv forskel på kurset, men et efteruddannelseskursus af den art, som har været genstand for undersøgelserne i dette studie, kan slet ikke stå alene. Det kan kun starte en proces, som kalder på yderligere bearbejdning. En undersøgelse af denne proces kræver yderligere forskning.

Referenceliste:

Becker J.P., Selter C. (1996): Elementary School Practices. Bishop A.J. and others.: *International Handbook of Mathematics Education*. [14], 511-564. Dordrecht, The Netherlands: Kluwer Academic Publisher.

Blomhøj M., Jensen T.H. (2002): *Developing mathematical modelling competence: Conceptual clarification and education planning*. Roskilde: Centre for research in learning Mathematics.

Blum W., Niss M. (1991): Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. In *Educational Studies in Mathematics, volume 22*. 22, 37-68. Dordrecht, the Netherlands: Kluwer Academic Publisher.

Chevallard Y. (1985): *La transposition didactique*. Grenoble: La pensé Sauvage.

Palm T., Burman L. (2004): Reality in mathematics assessment: An analysis of task-reality concordance in Finnish and Swedish national assessments. In *Nordisk matematikkdidaktikk, 9, no 3, October 2004*. 9, no 3, October 2004, 1-33. Göteborgs Universitet: NCM/NOMAD.

Watson A., Mason J. (1998): *Questions and Prompts for Mathematical Thinking*. Derby: ATM.